Summary. — The purpose of this paper is to clarify the origin of the projective invariance in dual-resonance models in terms of the Lorentz group. We find that the assignment of external particles to irreducible representations of $SL_2^c$ and the requirement that this group be an invariant group of the amplitude together with some physical requirements lead to the fully symmetric model proposed by Shapiro and Virasoro. Only the value $\alpha(0) = 2$ for the trajectory intercept is possible.

Introduction.

Much attention has been recently devoted to $N$-particle factorizable dual resonance models (1) (DRM). The main interest of these models is based on the fact that it is possible that an amplitude embodying in a consistent way duality and factorization can be a starting point for the construction of the

(1) For a general review, see, for example, G. Veneziano: Lecture Notes for the International School of Subnuclear Physics (Erice, 1970).
physical $N$-hadron amplitude. However, many fundamental features of DRM have not yet received a satisfactory understanding. In particular, the invariance properties (2) of DRM under the (real or complex) projective group are, at present, a mathematical way of imposing duality rather than a physically meaningful requirement. Nevertheless, the crucial role of projective invariance in DRM gives a strong indication that a deep physical meaning could underlie this group structure. Various attempts have been made in this direction. SUSSKIND (2), starting from a harmonic-oscillator model of hadrons including higher harmonics, has been able to construct the dual model proposed by VENENZIANO (4).

A more ambitious attempt in this direction has been performed by DOMOKOS et al. (5-7) who have proposed to study the restrictions imposed on the scattering amplitude by the following assumptions:

a) Lorentz invariance,

b) assignment of the external particles to irreducible representations of $SL_2\sigma$,

c) analyticity in the labels of these representations: « Lorentz quantum number ».

We shall refer to b) and c) briefly as « full Reggeization » hypothesis. The analysis of ref. (7) indicates that the projective invariant structure of DRM can be understood in terms of Lorentz invariance and full Reggeization. The purpose of this paper is to impose these requirements to their full extent and to implement them with general consistency conditions in order to see what limitations they put on the scattering amplitude of an arbitrary number of scalar particles.

We think that this analysis can throw some light on the question of whether the identity of the Lorentz group with the projective group related to DRM is a mathematical accident or reflects a deeper connection. We shall recall, for the reader's convenience some technical points of ref. (7).

The usual way to construct a Lorentz-invariant amplitude is to introduce a coupling scheme based on treelike diagrams (8). This procedure consists

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