The Physical Mass, the Renormalization Group and the Mass Generation (*) (**) 

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Summary. — Using the BPHZ renormalization procedure in its original form, we present an alternative possibility to define the physical mass within the framework of the massive $\phi^4$-model. It is shown that this mass satisfies the homogeneous renormalization group equation. We also deal with the problem of vanishing-mass parameters. The compatibility of the perturbative solution for the physical mass in the lowest-order approximation with the exact solution of the renormalization group equation is shown. The problem of spontaneous mass generation is also mentioned. 

1. – Introduction. 

The purpose of this short note is to make some comments concerning physical masses in the BPHZ scheme (1). As a testing ground we use the scalar $g\phi^4$-model. We assume the validity of the perturbation theory in the framework of the regulator-free version of the BPHZ (1) in the small-coupling limit. Within this perturbative realization of the model we use a definition of the physical mass which is ordinarily not used in the BPHZ renormalization scheme. 

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In sect. 2 we present this definition. The starting point is to consider the massive $g\varphi^4$-model characterized by one unphysical mass parameter $m^2$ (*) and the renormalized finite coupling constant $g$. It is also essential to choose two physical on-shell normalization conditions in order to require that the full renormalized propagator behaves like the free-field propagator. However, this is not enough to fix the physical mass and the corresponding two finite counterterms at each order of any perturbative calculation. Therefore, one needs one further condition. The end of sect. 2 is devoted to show that our physical mass of the model satisfies a homogeneous renormalization group equation. In addition, we also present the exact solution of this homogeneous differential equation which is useful for other purposes.

The aim of sect. 3 is to sketch the limit of vanishing-mass parameters. Due to the singularities of the propagator which one encounters in the limit $m^2 \to 0$, one has to use some off-shell normalization conditions. This limit can be investigated in a twofold manner. One possibility is to study directly the explicit expression of the physical mass. In a lowest-order approximation one finds that $m_p^2$ tends to zero as $m^2$ vanishes. The other alternative possibility makes use of the exact solution of the homogeneous renormalization group equation for the physical mass. The main ingredient in this step of our investigation is the fact that the $g\varphi^4$-model is an infra-red free-field theoretic model. This allows a comparison of the perturbative solution for the physical mass with its exact solution coming from the homogeneous renormalization group equation. The results depend very strongly on the off-shell normalization conditions chosen.

Furthermore, we believe that one cannot expect spontaneous generation of mass in the limit $m^2 \to 0$.

2. – On the definition of the physical mass: the renormalization group equation.

Usually, in applying the regulator-free scheme of BPHZ, one starts for the massive $\varphi^4$-model with the following effective Lagrangian (1):

\begin{equation}
\mathcal{L} = \frac{1}{2}(1 + b) \partial_\mu \varphi \partial^\mu \varphi + \frac{1}{4}(a - m^2)\varphi^2 + \frac{1}{4}(c - g)\varphi^4,
\end{equation}

where $a$, $b$ and $c$ are each a power series in the renormalized coupling constant $g$. These finite counterterms are defined implicitly by some normalization conditions. The parameter $m$ may be regarded as the physical mass of the particle described asymptotically by $\varphi$. In this approach $m$ is an independent parameter of the model.

(*) $m^2$ is the value in the tree approximation.