Summary. — It is argued that a recently proposed condition for colour confinement, based on BRS algebra and renormalization group, leads to a $k^{-4}$ infra-red behaviour for the structure function of the transverse gluon propagator. The conclusion holds if the number of quark flavours does not exceed nine.

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1. — Introduction.

The analytic properties and the asymptotic expression of the transverse gluon propagator have been thoroughly analysed in the literature (1,2). Because of asymptotic freedom, it is possible to derive an unsubtracted Lehmann representation for the structure function $D$ of the transverse gluon propagator. The sign and the asymptotic behaviour of the discontinuity of $D$, along the positive $k^2$-axis, depends on the sign of the quantity

$$\gamma_s = -\frac{1}{32\pi^2} \left( 13 - \frac{4}{3} N_f \right),$$

the lowest-order coefficient of the anomalous dimension for the gluon field

in the Landau gauge. Here, and in the following, the colour gauge group is $SU_3$ and all quarks are in the fundamental representation; $N_f$ denotes the number of quark flavours.

In the Landau gauge, if $\gamma_0/\beta_0 > 0$ ($\gamma_0 < 9$), the structure function $D$ satisfies a superconvergence relation. The state space has an indefinite metric and the spectral function $\varphi$, $\pi q = \text{Im} D$, needs not vanish.

These results can be extended (2) to the projected gluon propagator whose absorptive part contains all the contributions from positive-norm states. In this case the superconvergence relation, and hence the condition $N_f < 9$, is clearly inconsistent, unless the projected spectral function vanishes.

This possibility has been exploited and related to a condition for colour confinement based on BRS algebra (3). If, indeed, the projected gluon propagator vanishes, only the doublet part of the gluon field survives and single-gluon states are confined. According to ref. (4), this implies colour confinement for $N_f < 9$ but does not settle the matter when the number of quarks flavours exceeds nine.

Usually, the concept of confinement is associated with the nonperturbative long-distance structure of the theory. In dynamical approximation schemes of QCD, confinement is realized if the transverse gluon propagator has a strong infra-red singularity, its structure function $D$ behaving as $k^{-4}$ (4). In the approach of ref. (7), only the small-distance behaviour of the propagator is important. The low-energy properties of the gluon field appear only implicitly in the spectral representation or, in the case of the above confinement criterion, in the superconvergence relation. Since the result comes merely from renormalization group methods and analyticity, it seems that no correlation can be found with the dynamical condition on the infra-red singularity.

In this paper we show that, under a suitable hypothesis, the two pictures of confinement are connected. We will not resort, for the proof, to the projection of the state-vector space into a subspace of positive-definite metric but retain the other general postulates of ref. (2). We will also discuss the gauge dependence of the gluon propagator and corroborate the starting hypothesis with physical arguments.

In sect. 2, we briefly summarize the main results of recent papers (5-7) on the connection, induced by the renormalization group, between the infra-red region and the $g^2$ dependence in the weak-coupling limit. We discuss also the

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