Foundations of Basic Geometry

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The basic notions of length, area and volume were not alien to the prehistoric civilizations. The pyramids, palaces and great baths built more than 4000 years ago provide ample evidence. We begin our investigation of geometry with a discussion of areas of simple geometric objects.

1. Areas of Rectangles and Triangles

A rectangle is a plane figure formed by four straight lines meeting at right angles as shown in Figure 1.

![Figure 1](image)

We define its area as:

\[ \text{area of rect ABCD} = \text{base} \times \text{height} = bh. \]

This is the most obvious way to define area for if we double the base, the area should double, or if we triple the height the area should also triple. In modern language, we say that the area is linear in each variable \( b \) and \( h \). Up to a constant \( c > 0 \), which depends on the units of measurements, this is the only definition of the area if we want it to have these obvious properties.

From this definition we can show that the area of a triangle is given by the formula
area of triangle $= \frac{1}{2} (\text{base} \times \text{height})$.

We consider three possible cases.

1) The easiest case is that of a right triangle $\triangle ABC$ ($B = 90^\circ$). Its area is actually half the area of the rectangle $ABCD$ (Figure 2).

Thus

$$\text{area of } \triangle ABC = \frac{1}{2} (\text{area of the rectangle } ABCD)$$

$$= \frac{1}{2} (\text{base} \times \text{height}).$$

2) To compute the area of an acute triangle $\triangle ABC$, we begin by dividing the triangle into two right triangles as shown in (Figure 3).