A NODAL BASIS OF $C^\mu$–RATIONAL SPLINE FUNCTIONS ON TRIANGULATIONS

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Received June 29, 1992

Abstract

Let $\tau$ be a triangulation of a polygonal domain $D \subset \mathbb{R}^2$ with vertices $V = \{v_i : 1 \leq i \leq N_v\}$ and $RS^k(D, \tau) = \{u \in C^k(D) : \forall T \in \tau, u|_T \text{ is a rational function}\}$. The purpose of this paper is to study the existence and construction of $C^\mu$–rational spline functions on any triangulation $\tau$ for CAGD. The Hermite problem $H^k(V, U) = \{\text{find } u \in U : D^s u(v) = D^s f(v), |s| \leq \mu\}$ is solved by the generalized wedge function method in rational spline function family, i.e. $U = RS^k$. This solution needs only the knowledge of partial derivatives of order $\leq \mu$ at $v$. The explicit representations of all $C^\mu$–GWF (generalized wedge functions) and the interpolating operator with degree of precision at least $2\mu + 1$ for any triangulation are given.

1. Introduction

Let $\tau$ be a triangulation of a bounded polygonal domain $D \subset \mathbb{R}^2$, with vertex set $V = \{v_i : 1 \leq i \leq N_v\}$, and let us denote the grid–segments of $\tau$ by the symbols $\Gamma_i (s = 1, 2, \ldots, N_v)$. We define $RS^k(D, \tau) = \{u \in C^k(D) : \forall T \in \tau, u|_T \text{ is a rational function}\}$ and $S^k(D, \tau) = \{s \in C^k(D) : \forall T \in \tau, s|_T \in P_k\}$, where $P_k = \{\text{bivariate polynomials of total degree } \leq k\}$. Let $f \in C^r(D), r \geq \mu$, and $U$ be subset of $C^\mu(D)$. For a given $f(x,y) \in C^r(D), r \geq \mu$ and $U$ subset of $C^\mu(D)$, consider the following Hermite interpolation problem:

$$H^k(V, U) = \{\text{find } s \in U : D^s s(v_i) = D^s f(v_i), i = 1, \ldots, N_v, |s| \leq k\}.$$
subtriangulation (cf. [13][18]) or by investigating several special rational functions (cf. [2]). When $\mu = 1$ and $\mu = 2$, the authors also achieved the solution of the above problem via the generalized wedge function method. For the case of a general $n$, it seems to be very interesting to investigate furtherly the structure of $C^n$-rational spline functions.

The purpose of this paper is to develop a technique of rational spline function by GWF method and to investigate $C^n$-rational spline function for computer aided geometric design. We will give the method for constructing the explicit representations of $C^n$-generalized wedge functions for any triangle and the interpolating operator which has $2\mu + 1$ degree of precision.

2. Notation and Definitions

The two rational functions

$$R_1(x, y) = \frac{P_1(x, y)}{Q_1(x, y)}, \quad R_2(x, y) = \frac{P_2(x, y)}{Q_2(x, y)}$$

are identical if there is a constant $a \neq 0$ such that $P_1(x, y) = aP_2(x, y)$, $Q_1(x, y) = aQ_2(x, y)$.

The two rational functions in (1) are equivalent, if

$$P_1(x, y) \cdot Q_2(x, y) \equiv P_2(x, y) \cdot Q_1(x, y).$$

It is easy to verify that $R_1(x, y)$ and $R_2(x, y)$ are equivalent if and only if the irreducible rational representations of them are identical. In what follows, we shall regard two rational functions to be the same in the sense that equivalent.

The pathwork functions, denoted by $u(x, y)$, are defined over triangulation of $\tau$ of polygonal domain $D$ by specification of values (or derivative values), $u_q$, at all triangular element nodes and of associated basis (generalized wedge) functions $W_{q}^{T_i}$ on its triangular $T_i \in \tau$.

$$u(x, y) = \sum_{\text{all } q \text{ on element } T_i} u_q \cdot W_{q}^{T_i}(x, y), \quad (x, y) \in T_i.$$  

The family $\{W_{q}^{T_i}(x, y)\}$ has degree of precision at least $k$ iff

- for $u(x, y)$, any polynomial of maximal degree less than $k + 1$,

$$u(x, y) = \sum_{\text{all } q \text{ on element } T_i} u_q \cdot W_{q}^{T_i}(x, y), \quad (x, y) \in T_i,$$

- above equation is not true for $u(x, y)$ equal to at least one polynomial of degree $k + 1$.

- The aim of this paper is mainly to construct the basis $\{W_{q}^{T_i}(x, y)\}$ such that (2) has