Effect of aspect ratio on the meridional circulation of a rotating homogeneous fluid

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Abstract. The effect of aspect ratio on the meridional circulation of a homogeneous fluid is analyzed. Aspect ratio is allowed to range between zero and unity. Relationships between possible horizontal and vertical length scales are obtained by length scale analysis as well as by solving an idealized problem. It is found that when $E^{1/2} \ll Z \ll E^{1/2}/\delta$, where $E$ is the Ekman number, the stream lines are closely packed near the sidewall within a thickness of $O(E^{1/2})$. The effect of stratification is unimportant within this depth range. In the depth range $E^{1/2}/\delta \ll Z \ll 1/\delta$ the vertical boundary layer in which the streamlines are packed is of $O(EZ\delta)$. When $Z \gg 1/\delta$ it is shown that the circulation decays algebraically with depth as $1/Z$.

Keywords. Aspect ratio; meridional circulation; homogeneous fluid.

List of symbols:

- $u, v, w$ Velocity components
- $\phi$ Stream function
- $p$ Pressure
- $H, L$ Characteristic dimensions in vertical and horizontal directions
- $\delta$ aspect ratio
- $A_v, A_h$ Vertical and horizontal eddy coefficients of momentum
- $E_v, E_h$ Vertical and horizontal Ekman numbers
- $h, l$ Vertical and horizontal length scales
- $\xi$ Vertical distance
- $m$ Fourier transform variable
- $f$ Coriolis parameter.

1. Introduction

Analysis of sidewall friction boundary layers has assumed considerable importance since these layers play a significant part in ocean circulation, spin-up problems etc.
Stewartson [6] considered steady linear axisymmetric flow of a homogeneous fluid in a rotating cylindrical container with unit aspect ratio and found that the meridional circulation of fluid driven by the Ekman layers is completed within two sidewall layers, now known as Stewartson layers of thickness $E^{1/4}$ and $E^{1/3}$ where $E = \nu/\Omega L^2$ is the Ekman number. Pedlosky [4] and Durance and Johnson [3] analyzed the upwelling boundary layers whose axial scale is $O(1)$ for a linear and homogeneous oceanic model with $\beta$-plane approximation. While Pedlosky’s analysis spanned the range for the aspect ratio $\delta \ll E^{1/2}$, the analysis of Durance and Johnson was for the range $\delta \gg E^{1/2}$. The aim of this paper is to understand the role of various horizontal and vertical length scales associated with meridional circulation in a rotating homogeneous fluid with a constant Coriolis parameter for $0 \ll \delta < O(1)$.

The main motivation of this work is to elucidate the effect of aspect ratio on the various length scales and circulation pattern in a rotating hydrodynamic flow and to draw comparisons with other studies wherever possible. To keep the model more akin to an oceanic model, our mathematical analysis closely follows that of Blumsack [2] who had analyzed the transverse circulation near a coast in a rotating stratified fluid for $\delta^2 \ll S \ll 1$ where $S$ is the stratification parameter. Since Blumsack’s work was concerned with a stratified fluid and $S \gg \delta^2$ the effect of aspect ratio on the length scales and circulation pattern has not received considerable attention.

2. Governing equations

Following Blumsack [2], we assume that the perturbation state is independent of $y$, the alongshore coordinate and all dependent variables are functions of the offshore coordinate $x$ and the vertical coordinate $z$. Then the linear steady state equations in nondimensional form (for example see [2,4]) are,

$$-
u = -\frac{\partial P}{\partial x} + \frac{E_V}{2} \frac{\partial^2 u}{\partial z^2} + \frac{E_H}{2} \frac{\partial^2 u}{\partial x^2}$$

$$u = \frac{E_V}{2} \frac{\partial^2 v}{\partial z^2} + \frac{E_H}{2} \frac{\partial^2 v}{\partial x^2}$$

$$\omega = \frac{\partial P}{\partial z} + \delta^2 \left[ \frac{E_V}{2} \frac{\partial^2 w}{\partial z^2} + \frac{E_H}{2} \frac{\partial^2 w}{\partial x^2} \right]$$

$$\omega = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial z}$$

Here $\delta = \text{vertical length scale divided by horizontal length scale} = H/L$ is an aspect ratio; $u, v, w$ are velocity components, $P$ is the pressure. $E_V = 2A_V/fH^2$ and $E_H = 2A_H/fL^2$ are the vertical and horizontal Ekman numbers where $A_V$ and $A_H$ are eddy coefficients of momentum and $f$ is the Coriolis parameter.