Application of Weibull model for redefined significant wave height distributions

G MURALEEDHARAN, N UNNIKRISHNAN NAIR* and P G KURUP

Department of Physical Oceanography, Cochin University of Science and Technology, Fine Arts Avenue, Cochin 682 016, Kerala, India
*Department of Statistics, Cochin University of Science and Technology, Cochin 682 022, Kerala, India

It is well accepted that the parent distribution for individual ocean wave heights follows the Weibull model. However this model does not simulate significant wave height which is the average of the highest one-third of some 'n' (n-varies) wave heights in a wave record. It is now proposed to redefine significant wave height as average of the highest one-third of a constant number (n-constant, say, n = 100) of consecutive individual wave heights. The Weibull model is suggested for simulating redefined significant wave height distribution by the method of characteristic function. An empirical support of 100.00% is established by $\chi^2$-test at 0.05 level of significance for 3 sets of data at 0900, 1200 and 1500 hrs at Valiathura, Kerala coast. Parametric relations have been derived for the redefined significant wave height parameters such as mean, maximum one-third average, extreme wave heights, return periods of an extreme wave height and the probability of realising an extreme wave height in a time less than the designated return period.

1. Introduction

Detailed information on wave climatology is essential for all maritime activities including construction of coastal and offshore structures, harbours, offshore oil exploration and shipping activities (Draper 1973). Efficacy of marine structures and their cost analysis require a fairly good estimate of wave conditions (Ploeg 1968; Draper 1970). Often information on freak ocean waves and extreme wave conditions is needed in the design of offshore structures (Draper 1964; Thom 1971; Draper 1973). The only way to obtain reliable information on wave climate is to study them instrumentally and theoretically in all geographical areas.

Statistically meaningful understanding of random phenomena as waves requires a definite system of analyses through appropriate statistical techniques (Gouveia and Mahadevan 1983). The important statistical functions used to describe the basic properties of such random processes are

- probability distributions
- mean values
- auto correlation functions and
- spectral density functions.

Muraleedharan (1991) tried to identify the long-term wave height distribution pattern by probability density functions and derived therefrom parametric relations of certain important wave statistics. After bringing logical and experimental support for the validation of using the probability density functions for modelling waves, the wave statistics have been computed from visual wave observations. They are then compared with recorded wave measurements and utilised to infer the wave climate along the south-west coast of India (NIO 1982; NPOL 1978).

2. Materials and methods

The probability distributions used to model long-term distributions of wave heights are generally the log-normal, exponential, Weibull and Gumbel distributions. Studies have shown that none is superior to the

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others (Dattatri 1981; Baba 1985). For deep water conditions, the ratio of mean wave height \( (H_{\text{mean}}) \) to depth \( (d) \) of wave recording station tends to 0 \( ((H_{\text{mean}}/d) \rightarrow 0) \) and the Gluhovski distribution assumes the form of Rayleigh distribution (Shahul Hameed and Baba 1985). The Rayleigh distribution has also been recommended for long-term wave height modelling by Dattatri et al (1976).

The theoretical supremacy of the Weibull model based on its shape parameter taking care of the sea conditions suitable for the Longuet-Higgins model (Longuet-Higgins 1952) and also for situations under more general conditions has been shown by Muraleedharan (1991) in terms of the intensity function for decaying of waves. The motivation for considering Weibull model for long-term distribution of wave heights has been established empirically (Muraleedharan et al 1993).

Significant wave height is not a single value but the average of some \( 'n' \) wave heights in a wave record \( (n \text{ varies}, \ t \text{ constant}) \). If the significant wave height is redefined as the average of the highest one-third of a constant number of consecutive wave heights \( (n \text{ constant, say, } n = 100, \ t \text{ varies}) \), and since the Weibull model is confirmed as the parent distribution for individual wave heights, the model is suggested for the simulation of the redefined significant wave height distribution by the method of characteristic function.

2.1 Characteristic function of Weibull model

The density function of Weibull model is given by

\[
 f(h)dh = \frac{(b/a)(h/a)^{b-1} \exp-(h/a)^b dh}{a, b > 0, h > 0},
\]

where ‘a’ is the scale parameter, ‘b’ is the shape parameter and ‘h’ is the variable. The characteristic function of Weibull model is given by

\[
 E(e^{ith}) = \int_0^\infty e^{ith} f(h)dh
 = \int_0^\infty e^{ith}(b/a)(h/a)^{b-1} \exp-(h/a)^b dh,
\]

where \( 't' \) is an arbitrary real constant.

\[
 = \frac{(b/a)\int_0^\infty (\coshth + isinht/h)(h/a)^{b-1} \exp-(h/a)^b dh}{(b/a)\int_0^\infty \coshth(h/a)^{b-1} \exp-(h/a)^b dh + \int_0^\infty isinht(h/a)^{b-1} \exp-(h/a)^b dh}. \tag{A}
\]

Integrating the first expression of (A) by parts,

\[
 (b/a)\int_0^\infty \coshth(h/a)^{b-1} \exp-(h/a)^b dh
 = (b/a)\left[ \coshth \int(h/a)^{b-1} \exp-(h/a)^b dh \right].
\]

Put \( (h/a)^b = y \)

\[
 = 1 - t \int \sinht \exp-(h/a)^b dh. \tag{1}
\]

Now integrating the second expression of (A) by parts

\[
 (b/a)\left[ \int isinht(h/a)^{b-1} \exp-(h/a)^b dh \right]
 = (ib/a)\left[ \sinht \int(h/a)^{b-1} \exp-(h/a)^b dh 
 - \int \left[ (d(sinht)/dh)(h/a)^{b-1} \exp-(h/a)^b dh \right] dh \right].
\]

Put \( (h/a)^b = y \)

\[
 = it \int \coshth \exp-(h/a)^b dh. \tag{2}
\]

Adding (1) and (2) \( \Rightarrow 1 - t \int \sinht \exp-(h/a)^b dh + it \int \coshth \exp-(h/a)^b dh \]

\[
 = 1 + it \int \left[ \sum((it)^r/r!)(\int_0^\infty h^r \exp-(h/a)^b dh \right];
 r = 0, 1, 2, \ldots, \infty
\]

put \( (h/a)^b = y \)

\[
 = 1 + (it/b) \left[ \sum((it)^r/r!)(\int_0^\infty y^{(r+1)/b-1} e^{-y} dy \right]
 \int_0^\infty e^{-y}y^{(r+1)/b-1} dy.
\]

is the well known gamma function.

\[
 \phi(t) = \left[ 1 + \sum((it)^{r+1}/r!)(a^{r+1}/b)\Gamma((r + 1)/b) \right] \Gamma((r + 1)/b)
\]

\( \phi(t) \) is the characteristic function of the Weibull model.

2.2 Sampling distribution for redefined significant wave heights

It is to be noted that the characteristic function of the sum of \( 'n' \) independent variables is the product of their C. F. This simple property enables us to find the sampling distribution of the significant wave heights. (Maurice et al 1964). If we have a sample of \( 'n' \) values from a population whose characteristic function is \( \phi(t) \), the characteristic function of their sum is \( \phi^n \).

The characteristic function of the Weibull model has been derived to be

\[
 \phi(t) = \left[ 1 + \sum((it)^{r+1}/r!)(a^{r+1}/b)\Gamma((r + 1)/b) \right] \Gamma((r + 1)/b).
\]

Therefore the characteristic function of the sum of \( 'n' \) values is given by

\[
 \phi^n(t) = \left[ 1 + \sum((it)^{r+1}/r!)(a^{r+1}/b)\Gamma((r + 1)/b) \right] \Gamma((r + 1)/b)
\]