Complete solution for a two-dimensional tanh-conductivity arc

G C DAS
Department of Applied Mathematics, Indian Institute of Science,
Bangalore 560 012
Present address: Indian Institute of Astrophysics, Bangalore 560 034

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Abstract. This paper is a sequel to an earlier work on the dynamics of a two-dimensional tanh-conductivity arc. We present here a complete solution, based on the Galerkin technique, for the characteristic variation of current-voltage through an arc. The present results are qualitatively very similar to those obtained in earlier theoretical and experimental studies. The solutions obtained are the modification of the basic equations, which has led to quantitative investigations of the present problem. The influence of various arc parameters on the arc formation is also discussed.

Keywords. Arc; Galerkin method; tanh-conductivity.

1. Introduction

In recent years, a great deal of interest has been focussed on the advantages of the dynamics of arc phenomena. Many investigators have tried to get an insight into the basic mechanism of arc discharges but most of them have chosen an ideal model for the sake of mathematical simplicity. In this paper we attempt to study the basic mechanism of arc discharges in a two-dimensional tanh-conductivity arc. Due to the inherent limitations of the numerical procedure, an earlier study (Ayyaswamy et al 1978) of a two-dimensional tanh-model arc lead to an incomplete solution of the current-voltage variation. Further study of the arc was necessitated by modifying the basic equations governing the arc dynamics. We consider here a two-dimensional planar arc between two planar electrodes. The mathematical formulation of the model is very similar to that for the discharge lamps presented by Waymouth (1971). With the knowledge gained in our earlier work (Ayyaswamy et al 1978) the basic equations are now modified. We also study the equal and unequal temperatures of the cathode and anode. The results obtained here are similar to those obtained in another experiment on arc discharge lamps (Waymouth 1971).

2. Mathematical formulation of the problem

We consider a simplified model of a planar arc between two planar electrodes. Figure 1 shows the geometry, boundary conditions and co-ordinate systems of the arc. In the absence of radiation and convection, the fundamental equation for the energy conservation in the arc is given by the Elenbass-Heller equation,

\[ \nabla (k \nabla T) = -JE, \]

(1)
where $k$ is the thermal conductivity; $T$ is the temperature; $J$ is the current and $E$ is the electric field. Equation (1) is supplemented by Ohm's law: $J = \sigma E$, where $\sigma$ is the electrical conductivity assumed to be a function expressed by a tanh-function proposed first by Whitman et al (1976). Further $\nabla \times E = 0$ i.e. $E = -\nabla \phi$, where $\phi$ is the electrostatic potential. We introduce the heat flux $S$ as

$$S(T) = \int_0^T k(T') \, dT'.$$

With the definition of $S$, the electrical conductivity $\sigma(S)$ is expressed as:

$$\sigma(S) = \frac{1}{2} \sigma_0 \{1 + \tanh [a(S - S_0)]\},$$

where $\sigma_0$ and $a$ are material constants. The basic equations are subjected to the following appropriate boundary conditions:

$$x = \pm L, \quad S = S_w, \quad \partial \phi / \partial x = 0,$$
$$y = \pm l/2, \quad S = S_{c,a} (x), \quad \phi = \phi_{c,a}.$$

The total current through the arc is given by

$$I = \int J (\hat{J} \, dA)$$

(where $dA$ is the cross-sectional area) which can be represented after straightforward manipulation as:

$$\bar{I} = -\frac{1}{2} \int_{-1}^{1} \left[ 1 + \tanh (\mu \, \overline{S}) \frac{\partial \overline{\phi}}{\partial y} \, d\overline{x} \right],$$