ISOMETRIC MOTIONS OF A RELATIVISTIC SIMPLE FLUID (*)

S. GIAMBÒ - G. VALENTI

Some properties characterising the isometric motion of a relativistic simple fluid are pointed out. Corresponding to several state equations, the barometric formulas of the fluid are obtained. Finally the particular case of an incompressible fluid, from the relativistic point of view [3], is considered.

1. Introduction.

In their previous work, H. Dehnen, O. Obregon and C.D. Ciubotariu have studied the isometric motions both for an incompressible fluid (without internal energy) [1] and for an isentropic, ideal fluid [2] in the context of the general relativity theory. They have thus established the respective barometric formulas.

But, theoretical studies of astrophysics show that, in most real cases, we must take phenomena of viscosity and heat conduction into consideration; and, moreover, in the specific case of adiabatic motions, we must also consider their dependence on the entropy.

Consequently it would be interesting to generalize, in the context of the general relativity, the previous results [1], [2], in order to characterize the barometric formulas in the above mentioned cases.

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In this paper we study the isometric motions of a relativistic simple fluid by considering, in this first approach, only the adiabatic case.

Starting from a consideration of the fluid in question in section 2 and of its evolution system, in section 3 we deduce the general properties of its isometric motions.

In section 4 we obtain the barometric formulas which we will further particularise in the case of an incompressible fluid in section 5.

Notation.

The space-time $V_4$ is a 4-dimensional differentiable manifold with a normal hyperbolic Riemannian metric $ds^2$ of a signature $+---$, expressible in the usual form

$$ds^2 = g_{\alpha\beta} dx^\alpha dx^\beta$$

in an admissible local coordinate system $x^\alpha$.

Let us suppose that the manifold $V_4$ as well the potentials $g_{\alpha\beta}$ are differentiable up to the order needed to justify the following calculations.

The 4-velocity is defined by $u^\alpha = dx^\alpha/ds$, so that:

$$u^\alpha u_\alpha = 1.$$  

In what follows, $\partial_\alpha$ and $\nabla_\alpha$ denote the ordinary and the covariant derivatives as defined by the metric.

2. Description of the fluid and its evolution system.

Later on, we assume a coordinate system such that the velocity of light in the vacuum is 1.

In a domain $D \subset V_4$, a simple fluid is described [3] by an energy-momentum tensor of the form:

$$T_{\alpha\beta} = \tau f u_\alpha u_\beta - p g_{\alpha\beta}$$