COEFFICIENTS OF MULTIVALENT SYMMETRIC FUNCTIONS OF BOUNDED BOUNDARY ROTATION

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In this paper the sharp coefficient estimate problem for the classes $C_p(\beta, m)$ and $V_p(k, m)$ of multivalent close-to-convex functions of order $\beta$ and multivalent functions of bounded boundary rotation of at most $k\pi$, whose functions are given by $m$-fold symmetric gap series, have been discussed respectively for $\beta \geq 1 - \frac{p}{m} > 0$ and $k \geq 2(m/p)$. Moreover, it is shown that every function in $V_p(k, m)$ are $p$-valent close-to-convex; hence $p$-valent; if $k < 2 \left(1 + \frac{m}{p}\right)$.

Key Words and phrases. $m$ fold gap series, $p$-valent close-to-convex functions of order $\beta$, multivalent functions of bounded boundary rotation, coefficient estimate etc.

1. Introduction.

Let $E$ denote the unit disc $\{z : |z| < 1\}$. For $p = 1, 2, 3, \ldots$, let $A_p$ denote the class of functions $f$ analytic in $E$ and given by

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the power series

\[ f(z) = z^p + \sum_{n=p+1}^{\infty} a_n z^n. \]

A function \( f \) in \( A_p \) is said to be \( p \)-valent starlike, written as \( f \in Stp \), if

\[ \Re \left\{ \frac{z f'(z)}{f(z)} \right\} > 0 \quad \text{for} \quad z \in E. \] [10]

We say that a function \( f \) is \( p \)-valently close-to-convex of order \( \beta \), write as \( f \in C_p(\beta) \), \( \beta > 0 \), if there exists a function \( s \) in \( Stp \) and a real number \( \alpha \) such that

\[ \arg e^{i\alpha} \frac{zf'(z)}{s(z)} < \beta \frac{\pi}{2}. \] [12]

Note that \( C_1(1) \) is the classical family of univalently close-to-convex functions introduced by Kaplan [3], \( C_p(1) \) is the family of \( p \)-valently close-to-convex functions studied by Livingston [6] and \( C_1(\beta) \) is the family of close-to-convex functions of order \( \beta \) studied by Pommerenke [9]. Finally, for \( k \geq 2 \), let \( V_p(k) \) denote the subclass of functions \( f \) in \( A_p \) for which there exists a locally univalent function \( g \) of boundary rotation at most \( k\pi \) [8] such that

\[ f'(z) = pz^{p-1}(g'(z))^p, \quad f(0) = 0, \quad \text{for} \quad z \in E. \] [13]

Note that \( V_p(k) \) is a subclass of the class of multivalent functions of bounded boundary rotation studied by Leach [5], viz: functions having \((p-1)\) critical points located only at the origin and \( V_1(k) \) is the class of locally univalent functions of boundary rotation at most \( k\pi \) [8].

For \( m = 1, 2, 3, \ldots \), let \( Stp(m) \), \( C_p(\beta, m) \) and \( V_p(k, m) \), respectively denote the subclasses of \( Stp \), \( C_p(\beta) \) and \( V_p(k) \) whose functions are given by the \( m \)-fold symmetric gap series

\[ f(z) = z^p + \sum_{k=1}^{\infty} a_k z^{mk+p}, \quad z \in E. \] [14]