RANDOM POINTS ASSOCIATED WITH RECTANGLES

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The study of the distribution and moments of the distance between random points within a rectangle or in two coplanar rectangles is required in a wide variety of fields. Formulae for the distributions and arbitrary moments of the distance between two random points associated with one or two rectangles in various situations are given here explicitly. These explicit formulae will be helpful to those who work in various applied areas for the computations required in their problems.

1. Introduction.

Borel (1925) seems to have been the first one to consider the distance between random points in specific elementary geometric figures such as triangles, squares and so on. The theorems on mean values and fixed points of Crofton (1877, 1885) are very general in nature and although they cover particular geometric figures such as one and two rectangles, explicit formulae for these specific situations are not given there. The first papers giving explicit formulae for the density and mean values of the distance between two random points within a rectangle, in adjacent squares and in squares having a common diagonal seem to be those of Ghosh (1943a, 1943b, 1951). Since then several papers have been written on this topic. Expected distance is dealt with by many authors, see for example Christofides and Eilon (1969), Alagar (1976), Daley (1976), Oser (1976), Vaughan (1976) and Hsu (1990).

Some of the practical situations where the expected distance between

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random points in the same or different rectangles is required are the following: For applications in sampling problems and agricultural experimentation see Ghosh (1949), for urban operations see Larson and Odoni (1981), for management and mathematical modelling problems see Eilon et. al (1971), for transportation problems, vehicle routing, dispatching of emergency vehicles in urban settings and random paths across a rectangle see for example Horowitz (1965), Stone (1981) and Vaughan (1981). Horowitz (1965) also mentions some of the applications of random distances in physics such as in measuring the length of the path of a gamma-ray to the wall of a nuclear reactor and the length of a sound ray in a room from one reflection to the next. Kuchel and Vaughan (1981) look at the length of chords in a square which is applicable to modern electronic digitizing pads. Vaughan (1976) examines engineer’s "route factor" (that is, the ratio of the average distance by a given route to the average direct distance) for rectangular routing between adjacent squares. Marsaglia et. al (1990) state that the study of distance between random points in a rectangle arise in physical chemistry, chemical physics, material science, operations research and population studies and give a convenient spline function notation for the density which is more suitable for computational purposes and computer graphics. Gaboune et. al (1993) deal with generalized distances such as Manhattan metric and Chebychev metric, besides Euclidean metric. General procedures for tackling expected Euclidean distance in convex bodies with particular reference to polygons may be found in Kendall and Moran (1963), Fairthorne (1965), Coleman (1969, 1973), Ruben (1978), Solomon (1978), and Sheng (1985), among others, from where one can derive the results for special situations but with great effort.

Our aim in the present paper is to give a large number of explicit formulae for the general moments and densities by using elementary methods and without using results from integral geometry, most of which are assumed to be new, (although particular cases are available in the literature) of the distance between two random points where the points could be inside a rectangle, on opposite sides of a rectangle, on adjacent sides of a rectangle, one on a corner and the other inside a rectangle, in two different but similarly oriented rectangles. Euclidean distance, Manhattan distance and Chebychev distance are considered with reference to the densities and general moments. These formulae are expected to come in handy for those who want to apply them to practical situations in various disciplines.

2. Two random points within a rectangle.

When two points are selected at random on a particular side or