THE QUASI-STATIC PROBLEM
FOR AN ELECTROMAGNETIC CONDUCTOR

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Viene studiato il problema quasi-statico per un sistema elettromagnetico con una condizione al contorno dissipativa e con memoria. Si dimostra che se il nucleo di memoria soddisfa un'opportuna restrizione, traduzione in termini analitici della dissipatività locale della frontiera, allora il problema quasi-statico ammette una ed una sola soluzione debole.

1. Introduction.

In this paper we are going to study the quasi-static problem for an electromagnetic field, which describes the behaviour of a "good" conductor, that is of a medium with a high but finite electric conductivity. Because of the dissipation on the boundary, the condition which describes such a behaviour must satisfy a relation which imposes a restriction on the relative memory kernel. The aim of this paper is to show how this restriction is important in the proof of an existence and uniqueness theorem.

Consider a simply connected domain (i.e. a domain such that every continuous closed surface can be deformed continuously until it has shrunk to a point) $\Omega \subset \mathbb{R}^3$ with a bounded boundary $\partial \Omega$ of class $C^2$; the evolution of the electromagnetic field in $Q = \Omega \times (-\infty, +\infty)$
is ruled by the well-known Maxwell’s equations:

\[
\begin{align*}
\frac{\partial D}{\partial t} - \nabla \times H &= -J \quad \nabla \cdot D = \rho \\
\frac{\partial B}{\partial t} + \nabla \times E &= I \quad \nabla \cdot B = 0
\end{align*}
\]

(1.1)

If we assume that the material is a perfect dielectric, then the electric intensity \( E \), the magnetic intensity \( H \), the electric flux density \( D \) and the magnetic flux density \( B \) are connected by the following constitutive equations

\[
\begin{align*}
D(x, t) &= \varepsilon E(x, t) \\
B(x, t) &= \mu H(x, t)
\end{align*}
\]

(1.2)

where \( \varepsilon, \mu \) are constant second-order tensors, while the charge density \( \rho \) is supposed to be zero and the magnetic and electric current densities \( I, J \) supplied only by external sources and therefore known.

The vector \( I \) is usually set equal to zero, which means that magnetic currents do not occur in nature, however we let for a non zero \( I \), because it might represent a forced electric displacement current.

Moreover we suppose that the boundary of the domain \( \Omega \) is realized by a “good” conductor, so the relation between the electric and magnetic intensity on \( \partial \Omega \), is described by (see [3]):

\[
E_\tau(x, t) = \alpha_0(x) H_\tau(x, t) \times n(x) + \int_0^{+\infty} \alpha(x, s) H_\tau(x, t - s) \times n(x) ds,
\]

(1.3)

where \( \alpha \in L^1(0, +\infty; L^\infty(\Omega)) \), \( \alpha_0 \geq 0, n \) is the unit outward normal to \( \partial \Omega \), whereas \( E_\tau \) and \( H_\tau \) represent respectively the tangential component of \( E \) and \( H \).

The boundary condition (1.3) is a generalization of the Schelkunoff-Graffi’s condition (see [4]); in fact when we are in presence of harmonic fields of frequency \( \omega \), i.e. when \( E(x, t) = \hat{E}(x, \omega) e^{i\omega t}, H(x, t) = \hat{H}(x, \omega) e^{i\omega t} \), from (1.3) we have:

\[
\hat{E}(x, \omega) = \left[ \alpha_0(x) + \int_0^{+\infty} e^{-i\omega s} \alpha(x, s) ds \right] \hat{H}(x, \omega) \times n(x),
\]