QUASI-K-COSYMPLECTIC SUBMERSIONS

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In this paper we study the Riemannian submersions between manifolds belonging to those classes of almost contact metric manifolds which can be assembled under the common denomination of quasi-K-cosymplectic manifolds.

0. Introduction.

In [3], we have examined the differential geometric properties of almost contact Riemannian submersions between almost contact metric manifolds (almost contact metric submersions).

In this article, we study the properties of almost contact Riemannian submersions between quasi-K-cosymplectic manifolds (quasi-K-cosymplectic submersions).

First of all we recall in §1 the definition and properties of an almost contact metric manifold and an almost contact metric submersion.

In §2, we want to examine the influence of the almost contact metric nature of the submersion on its configuration tensors.

In §3, we prove that the horizontal distribution of an almost contact metric submersion whose total space is cosymplectic is completely integrable.

The relations between the φ-sectional and φ-bisectional curvatures of the two manifolds of an almost contact metric submersion are studied in §4.

1. Preliminaries.

A \((2n+1)\)-dimensional real differentiable manifold \(M\) of class \(C^\infty\) is said to have a \((\varphi, \xi, \eta)\)-structure or an almost contact structure if it admits a field \(\varphi\)

AMS Subject Classification Scheme (1.979): 53C15, 53C40.
of endomorphism of the tangent spaces, a vector field $\xi$, and a 1-form $\eta$ satisfying

\begin{align}
(1.1) & \quad \eta(\xi) = 1, \\
(1.2) & \quad \psi^2 = -I + \eta \otimes \xi
\end{align}

where $I$ denote the identity transformation [6]. Then $\psi \cdot \xi = 0$ and $\eta \cdot \psi = 0$; moreover, the endomorphism $\psi$ has rank $2n$, [1].

If a manifold $M$ with a $(\phi, \xi, \eta)$-structure admits a Riemannian metric $g$ such that

\begin{align}
(1.3) & \quad g(\phi X, \phi Y) = g(X, Y) - \eta(X) \eta(Y),
\end{align}

where $X, Y \in \chi(M)$, then $M$ is said to have a $(\phi, \xi, \eta, g)$-structure or an almost contact metric structure and $g$ is called a compatible metric [6]. An immediate consequence is $\eta(X) = g(X, \xi)$. The 2-form $\Phi$ on $M$ defined by

\begin{align}
(1.4) & \quad \Phi(X, Y) = g(X, \phi Y)
\end{align}

is called the fundamental 2-form of the almost contact metric structure.

Let $M$ be a manifold with an almost contact structure $(\phi, \xi, \eta)$ and consider the manifold $M \times R$. We denote a vector field on $M \times R$ by $(X, a \frac{d}{dt})$ where $X$ is tangent to $M$, $t$ the coordinate of $R$ and $a$ is a $C^\infty$ function on $M \times R$. S. Sasaki and Y. Hatakeyama [7] define an almost complex structure $J$ on $M \times R$ by

\begin{align}
(1.5) & \quad J(X, a \frac{d}{dt}) = (\phi X - a \xi, \eta(X) \frac{d}{dt})
\end{align}

and they prove that $J$ is integrable if and only if

\begin{align}
(1.6) & \quad N + 2 \eta \otimes \xi = 0,
\end{align}

where $N$ is the Nijenhuis tensor of $\phi$.

Now, if $g$ is a Riemannian metric on the manifold $M$ with a $(\phi, \xi, \eta)$-structure, we define a Riemannian metric on $M \times R$ by

\begin{align}
(1.7) & \quad h((X, a \frac{d}{dt}), (Y, b \frac{d}{dt})) = g(X, Y) + ab,
\end{align}

then, the following conditions are equivalent ([5]):