THE EXTENDED BITSADZE-LAVREN'T'EV-TRICOMI
BOUNDARY VALUE PROBLEM

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F. G. Tricomi ([5], [6]) originated the theory of boundary value problems for mixed type equations by establishing the first mixed type equation known as the Tricomi equation

\[ y \cdot u_{xx} + u_y = 0 \]

which is hyperbolic for \( y < 0 \), elliptic for \( y > 0 \), and parabolic for \( y = 0 \) and then observed that this equation could be applied in Aerodynamics and in general in Fluid Dynamics (transonic flows). See: M. Cribario [1], G. Fichera [2], and our doctoral dissertation [4]. Then M. A. Lavrent'ev and A. V. Bitsadze [3] established together a new mixed type boundary value problem for the equation

\[ \text{sgn}(y) \cdot u_{xx} + u_y = 0 \]

where \( \text{sgn}(y) = 1 \) for \( y > 0 \), \( = -1 \) for \( y < 0 \), \( = 0 \) for \( y = 0 \), which involved the discontinuous coefficient \( K = \text{sgn}(y) \) of \( u_{xx} \) while in the case of Tricomi equation the corresponding coefficient \( T = y \) was continuous. In this paper we establish another mixed type boundary value problem for the extended Bitsadze-Lavrent'ev-Tricomi equation

\[ L \cdot u = \text{sgn}(y) \cdot u_{xx} + \text{sgn}(x) \cdot u_{yy} + r(x, y) \cdot u = f(x, y) \]

where both coefficients \( K = \text{sgn}(y) \), \( M = \text{sgn}(x) \) of \( u_{xx} \), \( u_{yy} \), respectively are discontinuous, \( r = r(x, y) \) is once continuously differentiable, \( f = f(x, y) \) continuous, and then we prove a uniqueness theorem for quasi-regular solutions.

The Extended Bitsadze-Lavrent’ev-Tricomi Problem

Consider equation

\[
L \cdot u = \text{sgn}(y) \cdot u_{xx} + \text{sgn}(x) \cdot u_{yy} + r(x, y) \cdot u = f(x, y)
\]

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in a bounded simply connected region \( G \subseteq \mathbb{R}^2 \) by the curves: A piecewise smooth curve \( g_0 \) lying in the region \( G_1: x > 0, y > 0 \) and intersecting the line \( y = 0 \) at the point \( B(x_b, 0), (x_b > 0) \), and the line \( x = 0 \) at the point \( C(0, y_c), (y_c > 0) \), a smooth curve \( g_2 \) through \( B \) meeting a characteristic \( s_1 \) of the equation (1) issued from \( A(0, 0) \) at the point \( P_1 \) in the region \( G_2: x > 0, y < 0 \), the curve \( g_1 \) consisting of the portion \( A P_1 \) of \( s_1 \), a smooth curve \( S_2 \) through \( C \) meeting a characteristic \( s_2 \) of the equation (1) issued from \( A(0, 0) \) at the point \( P_2 \) in the region \( G_3: x < 0, y > 0 \), and the curve \( S_1 \) consisting of the portion \( P_2 A \) of \( s_2 \) in the region \( G_3 \).

It is clear that we can consider equations

\begin{align*}
\text{(c1)} & \quad g_1: x = -y \text{ in } G_1, \\
\text{(c2)} & \quad g_2: x = x_b + k \cdot y \ (k \geq 1) \text{ in } G_2, \\
\text{(c3)} & \quad S_1: y = -x \text{ in } G_3, \\
\text{(c4)} & \quad S_2: y = y_c + h \cdot x \ (h \geq 1) \text{ in } G_1,
\end{align*}

such that \((c_1)\) and \((c_3)\) satisfy the characteristic equation

\[(2) \quad \text{sgn} \ (y) \cdot (d \ y)^2 + \text{sgn} \ (x) \cdot (d \ x)^2 = 0\]