THE EXTENDED BITSADZE-LAVRENT’EV-TRICOMI BOUNDARY VALUE PROBLEM

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F. G. Tricomi ([5], [6]) originated the theory of boundary value problems for mixed type equations by establishing the first mixed type equation known as the Tricomi equation

\[ y \cdot u_{ss} + u_{r} = 0 \]

which is hyperbolic for \( y < 0 \), elliptic for \( y > 0 \), and parabolic for \( y = 0 \) and then observed that this equation could be applied in Aerodynamics and in general in Fluid Dynamics (transonic flows). See: M. Cribario [1], G. Fichera [2], and our doctoral dissertation [4]. Then M. A. Lavrent’ev and A. V. Bitsadze [3] established together a new mixed type boundary value problem for the equation

\[ \text{sgn}(y) \cdot u_{ss} + u_{r} = 0 \]

where \( \text{sgn}(y) = 1 \) for \( y > 0 \), \( = -1 \) for \( y < 0 \), \( = 0 \) for \( y = 0 \), which involved the discontinuous coefficient \( K = \text{sgn}(y) \) of \( u_{ss} \) while in the case of Tricomi equation the corresponding coefficient \( T = y \) was continuous. In this paper we establish another mixed type boundary value problem for the extended Bitsadze-Lavrent’ev-Tricomi equation

\[ L u = \text{sgn}(y) \cdot u_{ss} + \text{sgn}(x) \cdot u_{rr} + r(x, y) \cdot u = f(x, y) \]

where both coefficients \( K = \text{sgn}(y) \), \( M = \text{sgn}(x) \) of \( u_{ss}, u_{rr} \), respectively are discontinuous, \( r = r(x, y) \) is once continuously differentiable, \( f = f(x, y) \) continuous, and then we prove a uniqueness theorem for quasi-regular solutions.

The Extended Bitsadze-Lavrent’ev-Tricomi Problem

Consider equation

(1) \[ L u = \text{sgn}(y) \cdot u_{ss} + \text{sgn}(x) \cdot u_{rr} + r(x, y) \cdot u = f(x, y) \]

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in a bounded simply connected region $G \subset \mathbb{R}^2$ by the curves: A piecewise smooth curve $g_0$ lying in the region $G_1: x>0, y>0$ and intersecting the line $y=0$ at the point $B(x_b, 0), (x_b>0)$, and the line $x=0$ at the point $C(0, y_c), (y_c>0)$, a smooth curve $g_2$ through $B$ meeting a characteristic $s_1$ of the equation (1) issued from $A(0,0)$ at the point $P_1$ in the region $G_2: x>0, y<0$, the curve $g_1$ consisting of the portion $A P_1$ of $s_1$, a smooth curve $S_2$ through $C$ meeting a characteristic $s_2$ of the equation (1) issued from $A(0,0)$ at the point $P_2$ in the region $G_3: x<0, y>0$, and the curve $S_1$ consisting of the portion $P_2 A$ of $s_2$ in the region $G_3$.

\[ g_1: x=-y \text{ in } G_2, \]
\[ g_2: x=x_b+k \cdot y \ (k \geq 1) \text{ in } G_2, \]
\[ S_1: y=-x \text{ in } G_3, \]
\[ S_2: y=y_c+h \cdot x \ (h \geq 1) \text{ in } G_3, \]

such that (c1) and (c3) satisfy the characteristic equation

\[ \text{sgn} (y) \cdot (dy)^2 + \text{sgn} (x) \cdot (dx)^2 = 0 \]