SPATIAL SUBMANIFOLD WITH QUASI-RECURRENT PAIRING OF NORMAL FIELDS IN AN \((n + 2)\)-DIMENSIONAL MINKOWSKI SPACE

DOROTEA NAITZA

Let \((\tilde{M}, \tilde{g})\) be a \(C^\infty\)-Minkowski space of dimension \(n + 2\) and let \(\mathcal{B}(\tilde{M})\) be the bundle of \textit{quasi orthonormal} frames \([1], [2]\) over \(\tilde{M}\). If \(T_p(\tilde{M})\) is the tangent space at \(p \in \tilde{M}\) one may split it as \(T_p(\tilde{M}) = \mathcal{H}_p \oplus S_p\) where \(\mathcal{H}_p\) and \(S_p\) are an \textit{hyperbolic} 2-plane and a \textit{space-like} \(n\)-plane.

Let \(\mathcal{A}_\xi = \{\xi_A; A = 1, \ldots, n + 2\} \in \mathcal{B}(\tilde{M})\) be such a frame and let \(\{\alpha^\xi\}\) be the dual \textit{co-frame}. Then \(\mathcal{H}_p\) is spanned by the two null (real) vectors of \(\mathcal{H}_\xi\), say \(\xi_a\) \((a = 1, n + 2)\).

If \(X \in \mathcal{H}_p\) is any hyperbolic vector field, we say, following R. Rosca \([3]\), that \(X\) defines a \textit{quasi-recurrent hyperbolic pairing} iff:

\[\nabla X = u \otimes X + v \otimes X', \quad \nabla X' = u' \otimes X' + v' \otimes X\]
where \( \langle X', X \rangle = 0, \ |X| = |X'|, \ X' \in H_p \) and \( u, v, u', v' \), are Pfaffians. Consider the immersion \( x : M \rightarrow \tilde{M} \) where \( M \) is a spatial submanifold of dimension \( n \) (i.e. integral maximal manifold of the involuted distribution defined by \( S_p \)).

Suppose that the mean curvature vector \( H \) associated with \( x \) is the induced value of \( X \). Then one finds that the following cases occurs:

(i) \( M \) is minimal (i.e. \( H \) vanishes);

(ii) \( M \) is total geodesic;

(iii) \( M \) is pseudo-minimal in the sens of R. Rosca [1], [4] (i.e. \( H \) is a null vector field) and contained in a hyperplane defined by \( H = C \) (\( C = \text{cost.} \)).

1. Let \( (\tilde{M}, \tilde{g}) \) be a \( C^\infty \)-Minkowski space of dimension \( n + 2 \) and let \( \mathcal{B}(\tilde{M}) \) be the quasi-orthonormal frames bundle over \( M \) [1], [2].

If \( \mathcal{R}_e = \{ \xi_A ; A = 1, \ldots, n + 2 \} \) is such a frame, suppose that \( \xi_a (a = 1, \ldots, n + 2) \) are null (real) vectors and \( \xi_r (r, s = 2, \ldots, n + 1) \) are space-like vectors (the metric \( \tilde{g} \) of \( \tilde{M} \) has a normal hyperbolic signature).

One has as is known [1], [2]

\[
\langle \xi_1, \xi_{n+2} \rangle = 1, \quad \langle \xi_r, \xi_s \rangle = -\delta_{rs}, \quad \langle \xi_r, \xi_A \rangle = 0, \quad A \neq r
\]

and one may split the tangent \( T_p(\tilde{M}) \) at \( p \in M \), as

\[
T_p(\tilde{M}) = \mathcal{H}_p \oplus S_p.
\]

In (1.2) \( \mathcal{H}_p = \{ \xi_a \} \) is a hyperbolic 2-plane and \( S_p = \{ \xi_A \} \) is a spatial n-plane. The frame \( \mathcal{R}_e = \{ \xi_A \} \) defines uniquely a dual co-frame \( \{ \tilde{\alpha}^A \} \) in the co-tangent space \( T^*_p(\tilde{M}) \) and viceversa. The line element \( dp \) of \( \tilde{M} \) (\( dp \) is a canonical vectorial 1-form in the Lie group \( G \)) is given by

\[
dp = \tilde{\alpha}^A \otimes \xi_A.
\]

By (1.1) the metric \( \tilde{g} \) in terms of \( \tilde{\alpha}^A \) is expressed by

\[
\tilde{g} = 2 \tilde{\alpha}^1 \otimes \tilde{\alpha}^{n+2} - \sum_r \tilde{\alpha}^r \otimes \tilde{\alpha}^r.
\]

The space \( \tilde{M} \) is structured by the connection

\[
\nabla \xi_A = \tilde{\alpha}^B \otimes \xi_B.
\]