ABSOLOUTELY CONTINUOUS GLOBAL SOLUTIONS OF THE INITIAL VALUE PROBLEM FOR NEUTRAL FUNCTIONAL DIFFERENTIAL EQUATIONS OF MIXED TYPE

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The paper considers initial value problems for functional differential equations of neutral type with deviations dependent on the required function and its derivative. By the help of the method of accretive operators natural sufficient conditions have been obtained which guarantee the existence and uniqueness of the absolutely continuous global solution of the problem set to be solved.

During the past years is a growing interest in the research on neutral type functional differential equations as well as a vigorous growth of publications. A detailed bibliography in this field can be found in the papers [1] - [5].

The aim of the present paper is to find natural sufficient conditions for the existence and uniqueness of the global absolutely continuous solution of the initial value problem for neutral type functional differential equations with deviations dependent on the required function and its derivative.

The paper contains two paragraphs. In the first paragraph employing the connection between the accretive and nonexpansive operators [6] - [10], some auxiliary propositions are proved. The second paragraph considers the following initial value problems for neutral type functional differential equations:

(1) \[ \dot{y}(t) = G(t, y(t), \Delta_1(t, y(t), \dot{y}(t))), \, \dot{\Delta}_2(t, y(t), \dot{y}(t))), \quad t > 0 \]
\[ y(t) = \tau(t), \quad \dot{y}(t) = \dot{\tau}(t), \quad t \leq 0 \]

(2) \[ \dot{y}(t) = G(t, y(t), \Delta_1(t, y(t), \dot{y}(t))), \, \dot{\Delta}_2(t)), \quad t > 0 \]
\[ y(t) = \tau(t), \quad \dot{y}(t) = \dot{\tau}(t), \quad t \leq 0 \]
where the unknown function \( y(t) \) assumes values in the Banach space \( B \) with norm \( \| \cdot \| \), while \( G(t, x, y, z) \), \( \tau(t) \), \( \Delta_1(t, u, v) \), \( \Delta_2(t, u, v) \), \( \Delta_2(t) \) are the given functions.

We have to note that depending on the choice of the functions \( \Delta_1(t, u, v) \), \( \Delta_2(t, u, v) \) and \( \Delta_2(t) \) the equations (1) and (2) might be retarded, advanced and, in general case of mixed type.

The initial value problem (2) is a particular case of the problem (1). It is considered separately because as it is concerned, the theorem for the existence and uniqueness is proved under more general assumptions.

Of special difficulty is the complicated structure of the transformed argument in the case when it depends on the required solution and its derivative. Then the operators defined by the right-hand side of the equations (1) and (2) do not possess the property of superpositional measurability. In the present paper this difficulty has been overcome at the expense of the requirement for Lipschitz continuity of these operators.

Note that in [11] have been considered only particular cases of the problem (1) by assumption

\[ \Delta_1(t, u, v) \leq t, \quad \Delta_2(t, u) \leq t. \]

§ 1. Some auxiliary propositions from the theory of the accretive operators.

Let \( E \) be a Banach space with norm \( \| \cdot \|_E \) and \( E^* \) be its conjugate space of norm \( \| \cdot \|_{E^*} \).

By \( \text{Dom} \, N \) denote the definition domain of the operator \( N \), and by \( \text{Range} \, N \) denote its range of values. By \( I \) denote the identity map.

Let \( \mathcal{J} : E \rightarrow 2^{E^*} \) be the duality map, i.e.

\[ \mathcal{J}(x) = \{ j \in E^* : \langle x, j \rangle = \| x \|_E^2 = \| j \|_{E^*}^2, \ x \in E \} \]

(see in detail [10]).

Definition 1. The operator \( N : \text{Dom} \, N \rightarrow E \) (\( \text{Dom} \, N \subset E \)) is called accretive if for any \( x, y \in \text{Dom} \, N \) and some \( j \in \mathcal{J}(x - y) \) the relation is fulfilled as follows:

\[ \langle N x - N y, j \rangle \geq 0. \]

Lemma 1.1 from [9] yields that the operator \( N \) is accretive iff the inequality

\[ \| (I + \lambda N) x - (I + \lambda N) y \|_E \geq \| x - y \|_E \]

holds for any \( x, y \in \text{Dom} \, N \) and \( \lambda > 0 \).