We give a complete decomposition of the space of curvature tensors with the symmetry properties as the curvature tensor associated with a symmetric connection of Riemannian manifold. We solve the problem under the action of $SO(n)$. The dimensions of the factors, the projections, their norms and the quadratic invariants of a curvature tensor are determined. Several applications for Riemannian manifolds with symmetric connection are given. The group of projective transformations of a Riemannian manifold and its subgroups are considered.

1. Introduction.

A. Let $M^n$ be any $n$-dimensional manifold with any symmetric connection $\nabla$; $R$ and $\rho$ are the curvature tensor and the Ricci tensor associated with $\nabla$. It is well-known that the Weyl projective curvature
The tensor has the form

\[ P(R)(X, Y)Z = R(X, Y)Z + \frac{1}{n^2 - 1} [\eta_\rho(X, Z) + \rho(Z, X)]Y - \]

\[ - \frac{1}{n^2 - 1} [\eta_\rho(Y, Z) + \rho(Z, Y)]X + \]

\[ + \frac{1}{n + 1} [\rho(X, Y) - \rho(Y, X)]Z \]

for \( n > 2 \), and for \( n = 2 \) we have

\[ P(R)(X, Y)Z = 0 \]

(see for example [27], [28], [36]) where \( X, Y, Z \ldots \in \mathcal{X}(M) \), the algebra of \( C^\infty \) vector fields on \( M \). \( P(R) \) is a tensor that is invariant with respect to each projective transformation of \( M \). \( P(R) \) characterizes a space of constant sectional curvature in very nice way: \( P(R) = 0 \) if and only if \( M^n (n > 2) \) is space of constant curvature (in that case \( R \) is the Riemannian curvature of \( M^n \)).

It is known that \( R \) satisfies the following algebraic properties:

(1.1) \[ R(X, Y) = -R(Y, X), \]

(1.2) \[ \sigma_{X,Y,Z} R(X, Y)Z = 0 \] (the first Bianchi identity).

\( P(R) \) satisfies these relations (1.1), (1.2) and also

(1.3) \[ \rho(P(R)) = 0. \]

**B.** Let \( V \) is an \( n \)-dimensional vector space and denote by \( \mathcal{R}(V) \) the vector space of curvature tensors. The development of the theory of the decomposition of \( \mathcal{R}(V) \) under the action of some group was initiated by Singer and Thorpe [30]. In this well-known paper the authors consider the vector space \( \mathcal{R}(V) \) consisting of all tensors having the same symmetries as the curvature tensor of a Riemannian manifold, including the first Bianchi identity (in particular for \( n = 4 \)) and they give a geometrical useful description of the splitting of \( \mathcal{R}(V) \) under