LEBESGUE DECOMPOSITION BY
\( \sigma \)-NULL-ADDITIVE SET
FUNCTIONS USING IDEALS

PAOLO DE LUCIA - ENDRE PAP

Using V. Ficker's and P. Capek's algebraic approach by ideals a
Lebesgue decomposition theorem for a wide class of non-additive set
functions called null-additive set functions is obtained. In special cases
decompositions theorems for \( \Theta \)-decomposable measures and \( \kappa \)-triangular
set functions are obtained.

1. Introduction.

Important Lebesgue decomposition type theorems were intensively
investigated in the measure theory for additive set functions: [1],
[2], [3], [4], [5], [6], [9], [11], [20].

The aim of this paper is to apply Ficker's [9] and Capek's [1],
by "null sets" on a wide class of non-additive set functions, so
called null-additive set functions [7], [17], [18], [19], [21]. This
class of set functions includes many important set functions as

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$k$-triangular set functions and $\oplus$-decomposable measures. They find many applications in practice: knowledge based systems, expert systems, fuzzy control, fuzzy sets, game theory, etc.

We introduce the notion of $\sigma$-null-additive set functions as generalization of the notion of $\sigma$-null-additive measure. Using this notion we obtain a Lebesgue decomposition type theorem for null-additive set functions.

As consequences we obtain the corresponding results for $\ominus$-decomposable measures (generalization of result from [14]) and $k$-triangular set functions.

2. $\sigma$-null-additive set functions.

Let $\mathcal{R}$ be a ring of subsets of the given set $X$, $m$ a set function, $m: \mathcal{R} \to [0, +\infty]$.

**DEFINITION 1.** We say that

a) $m$ is monotone if $A, B \in \mathcal{R}$ and $A \subset B$ imply $m(A) \leq m(B)$;

b) $m$ is continuous from below if for every increasing sequence $(A_i)$ of elements of $\mathcal{R}$ such that $\bigcup_{i \in \mathbb{N}} A_i \in \mathcal{R}$, we have

$$m\left(\bigcup_{i \in \mathbb{N}} A_i\right) = \lim_{i \to \infty} m(A_i);$$

c) $m$ is exhaustive if for every disjoint sequence $(A_i)$ of elements from $\mathcal{R}$ we have $\lim_{i \to \infty} m(A_i) = 0$;

d) $m$ is order continuous if for each sequence $(E_n)$ such that $E_n \searrow \emptyset$ we have

$$\lim_{n \to \infty} m(E_n \cap Y) = 0$$

uniformly with respect to $Y \in \mathcal{R}$;