POLYNOMIALS ON LOCALLY K-CONVEX SPACES

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Polynomially bornological, polynomially barrelled and polynomially infrabarrelled locally convex spaces over a non-Archimedean non-trivially valued complete field of characteristic zero are introduced and studied.

The notions of bornological, barrelled and infrabarrelled locally convex space over an ultrametric field have been considered by various authors (see [3], [10], [11] and [14], for instance), whereas the more general notions of polarly bornological, polarly barrelled and polarly infrabarrelled locally convex space over an ultrametric field have been discussed in [12]. In the present article, the classes of polynomially bornological, polynomially barrelled and polynomially infrabarrelled locally convex spaces over a non-Archimedean non-trivially valued complete field of characteristic zero are introduced, each of which being a more restricted class than the corresponding linear one. Necessary and sufficient conditions for sets of homogeneous polynomials to be equicontinuous and natural locally convex topologies on spaces of continuous homogeneous polynomials are investigated. The main results obtained here are polynomial analogues of certain significant results of the linear theory. It should also be mentioned that this article was written under the influence of [7], where the polynomial classification of complex locally convex spaces has been studied.
Throughout this article, we shall adopt the notation and terminology of [9] and [12]. $\mathbf{K}$ will represent a field of characteristic zero which is complete under the metric induced by its non-Archimedean non-trivial valuation $\|\|$, $\mathbb{N}$ the set of non-negative integers and $\mathbb{N}^* := \mathbb{N} - \{0\}$. $E$, $F$ and $G$ will represent Hausdorff locally $\mathbf{K}$-convex spaces in this case, we shall prefer the terminology of [14]; we shall also write locally $\mathbf{K}$-convex topology, $\mathbf{K}$-normed space, $\mathbf{K}$-Banach space). For each $m \in \mathbb{N}^*$, $\mathcal{P}(mE; F)$ will denote the $\mathbf{K}$-vector space of all continuous $m$-homogeneous polynomials from $E$ into $F$ ($P : E \to F$ is an $m$-homogeneous polynomial if there exists an $m$-linear mapping $A : E^m \to F$ such that $P(x) = A(x, \ldots, x)$ for all $x \in E$) and $\mathcal{P}(mE) := \mathcal{P}(mE; \mathbf{K}) \cdot \mathcal{P}(E; F)$ will denote the $\mathbf{K}$-vector space of all continuous polynomials from $E$ into $F$ ($P \in \mathcal{P}(E; F)$ if and only if there exist an $\ell \in \mathbb{N}$, a constant mapping $P_0 : E \to F$ and $P_m \in \mathcal{P}(mE; F)$ ($m = 1, \ldots, \ell$) such that $P = \sum_{m=0}^{\ell} P_m$).

The following result, a particular case of Theorem 1 of [9], will be frequently used in the sequel:

Let $m \in \mathbb{N}^*$, $E$, $F$ locally $\mathbf{K}$-convex spaces, and $\mathcal{C}$ a set of $m$-homogeneous polynomials from $E$ into $F$. Then the following conditions are equivalent:

a) $\mathcal{C}$ is equicontinuous;

b) $\mathcal{C}$ is equicontinuous at the origin of $E$;

c) For each continuous seminorm $q$ on $F$, there exist a non-void open subset $U$ of $E$ and an $M > 0$ such that $q(P(x)) \leq M$

for all $x \in U$ and $P \in \mathcal{C}$.

**DEFINITION 1.** A locally $\mathbf{K}$-convex $E$ is said to be polynomially bornological if, for every $m \in \mathbb{N}^*$ and for every polar $\mathbf{K}$-normed space $F$, each $m$-homogeneous polynomial from $E$ into $F$ which transforms bounded sets into bounded sets is continuous.