String cosmology in Bianchi I space-time

A BANERJEE, ABHIK KUMAR SANYAL and
SUBENOY CHAKRABORTY*
Department of Physics, *Department of Mathematics, Jadavpur University,
Calcutta 700032, India

MS received 1 April 1989; revised 21 July 1989

Abstract. Some cosmological solutions of massive strings are obtained in Bianchi I
space-time following the techniques used by Letelier and Stachel. A class of solutions
Corresponds to string cosmology associated with/without a magnetic field and the other
class consists of pure massive strings, obeying the Takabayashi equation of state \( \rho = (1 + W) \lambda \).

Keywords. String, space-time; magnetic field; cosmology; string-dust system; Bianchi I.

PACS Nos 98.90; 04.20

1. Introduction

At the early stage of the universe a phase transition occurs as the temperature lowers
below some critical temperature, and this can give rise to various topologically stable
defects of which strings are of most important whose world sheets are two dimensional
time-like surfaces (Kibble 1976). It has been noted (Kibble 1976) that the existence
of a large scale network of strings in the early universe does not contradict the
present-day observations of the universe and further the vacuum strings (Zeldovich
1980) can generate density fluctuations sufficient to explain the galaxy formation.

These strings have stress energy and they couple to the gravitational field so that
it may be interesting to study the gravitational effects which arise from strings.

This has been already done by several authors, (Vilenkin (1981), Gott (1985),
Garfinkle (1985), although the general relativistic treatment of strings was pioneered
by Letelier (1979) and Stachel (1980). Letelier (1983) presented some cosmological
solutions of massive strings in Bianchi I and Kantowski-Sachs space-time.

In geometrical string (massless) models, infinite number of degrees of freedom are
possessed by each string for which the end points move at the speed of light. This
problem is resolved by considering the realistic (massive) string model of Takabayashi
(1976). The energy-momentum tensor for the massive strings has been first formulated
by Letelier (1979), who considered the massive strings being formed by geometric
strings with particles attached along its extension. Its application to general relativity
first appeared in Letelier (1983), while Stachel (1980) considered massless strings. So
the total energy-momentum tensor for a cloud of massive strings can be written as

\[
T\mu{}^\nu = \rho \cdot V\mu \cdot V^\nu - \lambda \cdot x\mu \cdot x^\nu,
\]

where \( \rho \) is the rest energy density for a cloud of strings with particles attached to
them (p-strings). Thus we have

$$\rho = \rho_p + \lambda,$$

(2)

$\rho_p$ being the particle energy density and $\lambda$ being the string’s tension density. $V^\mu$ is the four velocity for the cloud of particles and $x^\mu$ is the four vector representing the string’s direction which essentially is the direction of anisotropy. Thus,

$$V^\mu V_\mu = -1 = -x_\mu x^\mu \text{ and } V_\mu x^\mu = 0,$$

(3)
in $(-, +, +, +)$ signature.

In the present paper, we study the string cosmology in axially symmetric Bianchi I space-time both in the presence and absence of a source-free magnetic field. The evolution of a string-dust system may be interesting in the presence of the cosmic magnetic field.

Melvin (1975) in his cosmological solution for dust and electromagnetic field argues that the presence of magnetic field is not as unrealistic as it appears to be, because for a large part of the history of evolution matter was highly ionized, and matter and field were smoothly coupled. Later during cooling as a result of expansion the ions combined to form neutral matter.

Since the number of unknown parameters appearing in the model exceeds the number of field equations by one, we require one more equation to find the exact solutions. This additional equation is an assumed relation between the metric coefficients in the first case, where the string universe is associated with a magnetic field. In the second case where there is no magnetic field we have assumed an equation of state $\rho = (1 + W)\lambda$, ($W > 0$, is a constant), which is known as Takabayashi string (or P-string).

Since there is no direct evidence of strings in the present day universe, we are, in general, interested in constructing models of a universe that evolves purely from the era dominated by either geometric string or massive strings and ends up in a particle dominated era with or without remnants of strings.

2. Einstein’s field equations.

We consider an axially symmetric Bianchi I metric, which is

$$ds^2 = -dt^2 + \exp (2x) dx^2 + \exp (2\beta)(dy^2 + dz^2),$$

(4)

where, $x = x(t)$ and $\beta = \beta(t)$.

Now, the energy-momentum tensor for the string dust with a magnetic field along the direction of the string, i.e. the $x$-direction is given by

$$T_{\mu}^{\nu} + E_{\mu}^{\nu} = \rho \cdot V_{\mu} V^{\nu} - \lambda \cdot x_\mu x^\nu + \frac{1}{4\pi} (F_{\mu} a F_{\nu} - \frac{1}{4} F_{a b} F_{\nu}^{\alpha \beta} \delta_{\mu}^{\alpha \beta}).$$

(5)

In the above, $T_{\mu}^{\nu}$ is the stress-energy tensor for a string-dust system, $E_{\mu}^{\nu}$ is that for the magnetic field and $F_{a b}$ is the electromagnetic field tensor. The other terms have already been explained in the previous section. In the co-moving co-ordinate system $V^\mu = \delta_{0}^\mu$ and

$$T_{0}^{0} = -\rho, \quad T_{1}^{1} = -\lambda, \quad T_{2}^{2} = T_{3}^{3} = 0 = T_{\mu}^{\nu} \text{ (for } \mu \neq \nu).$$

(6)