Gauge theories of weak and strong gravity

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Abstract. A review of some recent papers on gauge theories of weak and strong gravity is presented. For weak gravity, SL(2, C) gauge theory along with tetrad formulation is described which yields massless spin-2 gauge fields (quanta gravitons). Next a unified SL(2n, C) model is discussed along with Higgs fields. Its internal symmetry is SU(n). The free field solutions after symmetry breaking yield massless spin-1 (photons) and spin-2 (gravitons) gauge fields and also massive spin-1 and spin-2 bosons. The massive spin-2 gauge fields are responsible for short range superstrong gravity.

Higgs-fermion interaction can lead to baryon and lepton number non-conservation. The relationship of strong gravity with other forces is also briefly considered.

Keywords. Gauge theories; strong gravity; symmetry breaking; unification of forces.

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1. Introduction

It is well known that Einstein’s general theory of relativity (GTR) (in a sense phenomenological theory of gravity) is based on a geometrical approach. However, there are other ways of looking at gravity. Through the work of Utiyama (1956), Kibble (1961) and others (e.g. Eguchi et al 1980; Ivanenko and Sardanashivly 1983), it has been realised that gravitational field can be regarded as a non-Abelian gauge field of Yang-Mills (1954) type. This field is self-interacting and the equations are nonlinear. While some authors (Salam 1977) consider Einstein’s equations as gauge theory par excellence, others (e.g. Yang) contend that this is based on an unnatural interpretation of gauge fields. Nevertheless, the concept of gauge theory pervades GTR in the form of covariant derivatives.

In gauge theories, auxiliary fields appear when one considers invariance of the field equations under space-time dependent transformations. In GTR the role of these fields are played by Christoffel symbols for the group of coordinate transformations which are, in fact, space-time dependent. Over the last 15 years some important developments in the field of gravity have taken place. These are: (i) Superstrong short range gravity mediated by massive spin-two gauge bosons (Sivaram and Sinha 1979) in addition to weak gravity mediated by massless gravitons. (ii) Quantization of gravity: It had been hoped that like all other fields, gravity will also be quantized. Although there have been many interesting approaches to this greatest challenge of theoretical physics (for reviews see Isham et al 1981; Narlikar and Padmanabhan 1983), we do not have a solution to this problem as yet. (iii) Supergravity invokes supersymmetry between pairs of bosons and fermions, they being manifestations of the same super particle. Supersymmetry generalised to a local gauge invariance leads to a gravitational model as
a mixture of spin-2 (graviton) and spin 3/2 (gravitino) particles. Gravitational interaction emerges within the framework of a unified gauge theory. There is intense activity in this field with the hope of unifying all fundamental forces in nature with extended supergravity models and Kaluza-Klein supergravity (van Nienwenhuizen 1981; Duff et al. 1983). (iv) Another exciting development is the concept that gravity is not fundamental but is an induced effect arising from symmetry-breaking effect in quantum field theory. There is great promise but considerable work is needed before a clear picture emerges (Adler 1982; Zee 1982; Pagels 1983).

In the present article, we shall be mainly concerned with gauge theory of superstrong gravity and weak gravity with their possible connection with other interactions in nature, namely, weak, electromagnetic and strong interactions. Although the initial motivation for superstrong short range gravity came from the existence of massive spin-2 f-mesons (Isham et al. 1971; Sivaram and Sinha 1973), it can be generalized to spin-2 gluons or gauge bosons. An appropriate gauge theory can permit the existence of both short range and infinite range gravity.

In what follows, we shall discuss a gauge theory formulation along with a concept of spontaneously broken symmetry (sbs) (Linde 1979) and Higgs (1966)-Kibble (1967) mechanism which will lead to the generation of massless and massive spin-1 and spin-2 gauge fields.

2. SL(2, C) gauge theory of weak gravity

Here we briefly outline an SL(2, C) gauge theory formulation of Einstein type theory of weak gravitation using the tetrad formalism and the concept of spontaneously broken symmetry (Sivaram and Sinha 1975; Dennis and Huang 1977). SL(2, C) group is homomorphic to the proper orthochronous Lorentz group. The tetrad formalism is ideally suited to display the gauge aspect of the theory. The special feature of the formalism is invariance under tetrad rotation. This corresponds to invariance under the gauge group SL(2, C) and is analogous to the invariance of the Yang-Mills (1954) fields under local isotopic spin rotation in SU(2).

In the tetrad formalism four reference vectors are constituted at each space-time point. This is in addition to the four-coordinate system. We shall denote the tetrad by

\[ t^a_{\mu} \quad (a = 0, 1, 2, 3 \text{ for tetrad components} \]
\[ = 0, 1, 2, 3 \text{ for local coordinate indices} \] (1)

and their inverses \( t^\mu_a \), which satisfy

\[ t^a_{\mu} t^b_{\nu} = \delta^a_c t^c_{\nu}, t^a_{\mu} t^b_{\mu} = \delta^b_a. \] (2)

The relationship between the Lorentz metric \( \eta_{ab} = \text{diagonal} \) (1, -1, -1, -1) and the metric \( g_{\mu\nu} \) of general relativity is then given by

\[ g_{\mu\nu} = t^a_{\mu} t^b_{\nu} \eta_{ab}. \] (3)

Now the constant Dirac matrices (on flat space-time) satisfy the relation

\[ \gamma_a \gamma_b + \gamma_b \gamma_a = 2 \eta_{ab}. \] (4)

One can define space-time dependent Dirac matrices