Nuclear compressibility and its effect on nuclear binding energies from the study of muonic atoms

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Abstract. The variation of nuclear parameter with mass number elicits information about nuclear compressibility. Analysis of muonic x-ray transitions provides an elegant method to investigate the behaviour of the nuclear parameter \( r_0 \). It is observed from the behaviour of \( r_0 \) that nuclei in the region \( A < 70 \) are highly compressible while those in the region \( A \approx 210 \) are almost incompressible. The behaviour of \( r_0 \) is incorporated into the semi-empirical mass formula through the Coulomb energy term. From the modified mass formula thus obtained binding energies of about 440 spherical nuclei have been calculated. The results suggest that nuclear compressibility imposes certain relationship between excess binding energies \( (E_{exp} - E_{cal}) \) and neutron, proton number. The present study also points out that shell effects exhibited by nuclear binding energies cannot be accounted for by simply varying the coefficients of the mass formula; on the other hand extra terms are necessary to explain them.

Keywords. Muonic atom; nuclear compressibility; nuclear binding energies; mass formula.

1. Introduction

It is well-known that the semi-empirical mass formula (equation (1)) reproduces only the main trend in the variation of nuclear binding energies with mass number \( A \) and nuclear charge \( Z \).

\[
E = a_v A - a_s A^{2/3} - a_c Z^2 A^{-1/3} - a_{sym} (N - Z)^2 A^{-1} + \delta a_{pair} A^{-1/2}.
\]  

In the above mass formula \( a_v, a_s, a_c, a_{sym} \) and \( a_{pair} \) are the coefficients of the volume energy, surface energy, coulomb energy, symmetry energy and pairing energy respectively. The sign of the pairing energy term is determined by the \( \delta \) value, which is +1 for even-even nuclei, 0 for odd nuclei and -1 for odd-odd nuclei. The mass formula is discussed in detail by Wapstra (1958). The quantity \( (E_{exp} - E_{cal}) \), called the excess binding energy, is known to exhibit particularly large deviations in the vicinity of magic numbers (Wapstra 1958). The observed discrepancies between the experimental and calculated binding energies \( (E_{exp} - E_{cal}) \) are attributed mainly to the inadequacies of the assumptions involved in the liquid drop model employed in developing (1).

Important assumptions mainly suspected are: (i) the statistical assumptions, especially in connection with light nuclei, (ii) the assumption of uniform density; (iii) the assumption that nuclei are incompressible; and (iv) absence of shell structure of nuclei. It is well established that because of assumption (i), the mass formula is not strictly valid for light nuclei. Assumptions (ii) and (iii) are too stringent and must be regarded with scepticism. It is also well-known that shell effects are too important to be ignored. Besides, the liquid drop model is inadequate to explain nuclear deformations. The
individual contributions from the above mentioned physical effects may be in the same or opposite direction and they may not be independent. Hence, the entire problem of apportioning the binding energies of nuclei to the various above mentioned physical effects is expected to be very complicated. In the present study a method of investigating the effect of nuclear compressibility on the nuclear binding energies is described.

2. Analysis of muonic x-rays—nuclear compressibility

In the nucleus, if the charge is assumed to be distributed uniformly within a sphere having a radius \( R = r_0 A^{1/3} \), the parameter \( r_0 \) would be directly proportional to the cube root of volume per nucleon or inversely to the cube root of average nucleon density. Although \( R \) represents the charge radius, it is reasonable to assume that \( r_0 \) represents average nucleon density, because the charge and mass distributions in a nucleus are nearly identical (Hill 1957). The \( A^{1/3} \) dependence of the radius \( R \) is well known and is established by a variety of experiments. However, in many investigations like scattering experiments, study of isotope and isotone shifts in optical spectra and in the evaluation of nuclear binding energies the \( r_0 \) value is assumed to be constant—each experiment assuming a different value. Since it is proportional to the average nucleon density, the constancy of \( r_0 \) invariably implies the acceptance of incompressibility of nuclear matter—a fact which is not strictly valid. In fact, many discrepancies in the interpretation of various experimental results are attributed to this assumption. For example, the isotope shifts predicted by the relation \( R = r_0 A^{1/3} \) are nearly twice the experimentally observed values (Devons and Duerdoth 1969). But, if the parameter \( r_0 \) is allowed to vary, better results could be obtained (Subba Rao and Kamal 1983).

Analysis of muonic x-ray transitions is an established technique to study nuclear structure and provides an elegant method to investigate the compressibility of nuclear matter. Although uniform model is not very realistic, under the assumption of uniform charge density analysis of muonic x-rays gets simplified and yields reasonably good results (Subba Rao and Kamal 1980). It is observed that from such an analysis the variation of parameter \( r_0 \) with mass number \( A \) can be obtained (Subba Rao and Kamal 1983).

The radial part of the Dirac equation for a two-component wave function with a large component \( g \) and a small component \( f \) is solved to obtain the energy levels of the muon in the electrostatic field of the nucleus.

\[
\begin{align*}
\frac{df}{dr} &= \frac{kf}{r} - \frac{1}{\hbar c} \{ W - V(r) - \mu c^2 \} g, \quad (2a) \\
\frac{dg}{dr} &= \frac{1}{\hbar c} \{ W - V(r) + \mu c^2 \} f - \frac{kg}{r}. \quad (2b)
\end{align*}
\]

In (2) \( V(r) \) is the potential generated by the uniform charge density and has the form of an harmonic oscillator potential within the radius \( R \):

\[
V(r) = - \frac{3Ze^2}{2R} + \frac{Ze^2}{2R^3} r^2, \quad r \leq R
\]

\[
= - \frac{Ze^2}{r}, \quad r \geq R. \quad (3)
\]

The energy levels obtained are then corrected for vacuum polarisation effect. The method of solving the Dirac equation and calculating the parameter \( r_0 \) is given in detail.