Quantization of Zero-Width Classical « Dual » Vortices (*).

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Summary. — We suggest that the recently proposed correspondence between zero-width classical vortices and relativistic « strings » is destroyed by field quantization.

The most elegant realization of dual theory is the semi-classical analogue model of relativistic « strings » that has been developed by many authors (**). In such an approach « strings » are fundamental geometric objects whose nature is not specified.

In an attempt to clarify some aspects of this approach several authors (2,3) have suggested that the relativistic « string » can be interpreted as a collective vortex mode of a local field theory. It is easily seen that classical local field theories in which there is spontaneous symmetry breaking permit static vortex line solutions (which can be considered as relativistic generalizations of the superfluid vortex solutions of an imperfect Bose gas (4)).

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.

(**) The reader is referred to ref. (1) for a short list of some of the most significant papers on this topic.


The major properties of such classical vortex solutions are that

i) the thickness of a vortex line (i.e. the area over which the energy density is significantly different from zero) is given by the Compton wavelengths of those local fields which acquire mass via the spontaneous symmetry breaking;

ii) in the limit of infinite masses the energy density of a vortex line tends to a two-dimensional $\delta$-function along the line;

iii) as a consequence of ii) the classical action governing the motion of a zero-width vortex is plausibly proportional to the area swept out by the vortex line in its motion. Since this is also the classical action of a relativistic string, at a classical level the correspondence between field theory vortices and strings is fairly well established.

The major problem with the field theory approach outlined above is that it is not obvious whether vortex line solutions have any stability in the infinite-mass (or coupling constant) limit, corresponding to the zero-thickness limit, in a second quantized theory. The authors of ref. (2) expected some difficulty in that this limit is "super quantum mechanical" in the sense that the typical classical action of the theory is arbitrarily small, compared to $\hbar$.

In this paper we re-examine the problem of quantization. Our conclusion is that in the thin vortex limit quantization destroys the correspondence between vortices and strings, in accord with the doubts of ref. (2).

At a heuristic level it is not hard to see how this comes about, and we shall sketch the main idea for the simplest model for which vortex solutions exist. We defer a more explicit approach to our conclusion to the Appendix of this paper.

We first make the comment that properties i) to iii) above are just the consequences of spontaneous symmetry breaking. The authors of ref. (2,3) discussed the symmetry breaking of a local gauge theory. The advantages of such a local gauge theory (corresponding more to the relativistic generalization of superconductivity than superfluidity) are that all fields become massive, and that the inclusion of magnetic monopoles enables us to consider finite open strings.

For our purposes it is sufficient to consider a theory with only global symmetry. In particular, in the Appendix we discuss canonical quantization in the Schrödinger picture, and the problems of canonical quantization in a local gauge theory are well known.

We consider the theory of a complex scalar field $\varphi$ with global $U(1)$-symmetry described by the Lagrangian density

\[ \mathcal{L}_\varphi = \partial_\mu \varphi^* \partial^\mu \varphi - \frac{m^2}{2\alpha^2} \left( \varphi^* \varphi - \frac{1}{2\alpha^2} \right)^2. \]