TRANSVERSALITY ON INFINITE DIMENSIONAL MANIFOLDS WITH CORNERS*

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In this paper we study transversality on infinite dimensional manifolds with corners. We deal with transversality, boundary-transversality and weak-boundary-transversality, characterize these three notions by means of the infinitesimal transversality, construct submanifolds as inverse image of submanifolds by transversal maps and prove parametrized theorems about the density of weak-boundary-transversality and boundary-transversality, which generalize the corresponding Abraham and Quinn theorems.

1. Introduction.

As it is well known, transversality theory is an important tool in Differential Topology. By means of these techniques we can, for instance, define algebraic invariants of the manifolds. (see [8]).

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The basic subjects to be studied are a local description of transversality, an infinitesimal description of transversality, construction of submanifolds as inverse images of submanifolds under transversal maps, density of parametrized transversality to a submanifold, density of parametrized transversality to a Fredholm map, density of the k-jet transversality and density of multi k-jet transversality. Classical results about these topics (finite dimensional manifolds without boundary) can be found in [7].

A first difficulty, in this study, concerns the infinite dimensional manifolds without boundary. Indeed, since differentiable maps preserve the sets which Lebesgue measure is zero, one can define measure zero sets in finite dimensional manifolds. The Sard density theorem asserts that the set of critical values of a differentiable map is a measure zero set and therefore, assuming suitable topological restrictions, one deduces that the set of regular values of a differentiable map is a dense set. The pattern used to define measure zero sets in \( \mathbb{R}^n \) can not be followed in the case of infinite dimensional manifolds. Nevertheless for Fredholm maps the set of regular values is a dense set. This generalization is due to S. Smale [15]. In this context Smale density theorem, Quinn density theorem, Abraham parametrized theorem of the density of the transversality, Quinn parametrized theorem of the density of the transversality, Thom theorem of the density of the k-jet transversality and Mather theorem of the density of the multi k-jet transversality are well known. (see [10]).

A second step of difficulty concerns the infinite dimensional manifolds with corners. In this paper we develop techniques which allow us to study these problems in a natural way.

In paragraph 2, we give known definitions and results as prerequisites. In paragraph 3, we give the definition of weak-boundary-transversality, in a coherent manner with the corners of the manifolds, we study also the relations with other definitions of transversality, characterize these definitions by means of infinitesimal transversality and construct manifolds as inverse image of submanifolds. In paragraph 4, we obtain parametrized theorems about the density of the weak-boundary-transversality and boundary-transversality which