A THIRD GENERATION SHALLOW WATER WAVE NUMERICAL MODEL—YE—WAM*

YIN Bao-shu (尹宝树), WANG Tao (王涛)
(Institute of Oceanology, Chinese Academy of Sciences, Qingdao 266071)
M. I. El-Sabh
(Department of Oceanography, University of Quebec at Rimouski, PQ, Canada G5L 3A1)

Received May 2, 1995; revision accepted Aug. 21, 1995

Abstract

This paper presents a third generation shallow water discrete spectral wave numerical model YE—WAM based on the spectral action balance equation. The model accounts for all relevant effects of currents on waves, including temporally and spatially varying depth and current induced refraction, straining and frequency shift and also explicitly takes into account all source terms, specially a depth-limited breaking dissipation. In addition, an energy forcing scheme is proposed and applied to the model's open boundaries to account for the propagation of swells into the study system. The upwind differencing scheme and a standard hybrid differencing scheme for the propagation term and a simple Euler method for the source terms are employed.

Key words: wave numerical model, shallow water, depth-limited dissipation.

INTRODUCTION

Since the pioneering paper of Geld et al. (1957), numerical wave prediction models have been formulated in terms of the basic transport equation for the two-dimensional wave spectrum. At present, the most advanced directional wave spectral model is the so-called third generation wave model-WAM model (WAMDI Group, 1988), which accounts for all processes of wave generation, dissipation and nonlinear wave-wave interactions explicitly.

Although these computationally complex directionally spectral models have achieved significant improvements in wave prediction, many uncertainties still remain. Many wind wave models are being used not only for marine forecasting and rational engineering design but also for understanding and verifying the mechanisms involved in wave evolution.

Almost all present wave prediction models do not incorporate wave-current interactions. Several numerical wave models have been developed for waves on currents (Chen and Wang, 1983; Holthuijsen, 1989). However, these models were developed for (quasi) stationary depths and currents for which changes of absolute frequency due to the unsteadiness of depth and currents can be neglected.

The present model YE—WAM accounts for all wave-current interaction processes. Also, to perfect the physics of the model, a depth-limited breaking mechanism in shallow seas has been taken into account. Thus, the YE—WAM model is a third generation shallow water discrete spectral wave numerical model considering temporally and spatially varying depths and currents and depth-induced breaking as well as traditional sources explicitly.

* Contribution No. 2811 from the Institute of Oceanology, Chinese Academy of Sciences.
Supported by the National Eighth-Five-Year Project D09920109 and Chinese Academy of Sciences and State Education Commission.
Wind waves are usually described with an energy density spectrum as a function of wave phase parameters such as the wave number $k$, the intrinsic or relative frequency $\sigma$, the absolute frequency $\omega$ and the direction $\theta$. In the linear theory for (quasi-) uniform surface gravity waves on slowly varying depth and currents (Mei, 1983), the wavenumber $k$ is related to the frequencies $\sigma$ and $\omega$ in the dispersion relation:

$$\sigma = \sqrt{gk\tanh d} = \omega - k \cdot \vec{u}$$

Wind wave propagation in changing water depth is (in linear wave theory) most conveniently described with the spectral action balance equation (Mei, 1983). In the present study, the YE-WAM model is based on the following discrete spectral action balance equation:

$$\frac{\partial N}{\partial t} + \nabla \cdot [(C_g + \vec{u})N] + \frac{\partial}{\partial \omega} [C_\omega N] + \frac{\partial}{\partial \theta} [C_\theta N] = \frac{S}{\sigma}$$

where $N(\omega, \theta, x, y, t)$ is the action density spectrum, the quantities $c$ represent propagation velocities in several spaces (see below) and $S$ represents the net source term for wave energy. The energy propagation velocity $C_g$ has the direction $\theta$, and source function terms $S(\omega, \theta, x, y, t)$ include:

1. Wind input $S_{in}$
2. Nonlinear resonant wave-wave interactions $S_{nl}$
3. Dissipation due to deep-water wave breaking $S_{db}$ (whitecapping)
4. Dissipation due to bottom friction $S_{bot}$
5. Depth-limited breaking dissipation in shallow water $S_{sh}$

The left hand side of Eq. (2) represents conservative action propagation. The first term of Eq. (2) is the local rate of change of the action density. The second term of Eq. (2) represents convection (rectilinear propagation) in the geographic (physical) space due to the wave energy propagation velocity $C_g$ and the mean current velocity $\vec{u}$. This term includes straining of the wave field due to spatial variation of $C_g + \vec{u}$. The third term of Eq. (2) represents shifting of the absolute frequency due to time variations in depth and currents with $C_\omega$. The fourth term of Eq. (2) represents refraction with $C_\theta$. The propagation speeds $C_g$, $C_\omega$ and $C_\theta$ are given by (Mei, 1983):

$$C_g = \frac{\sigma}{k} \left( \frac{1}{2} + \frac{kd}{\sinh 2kd} \right)$$

$$C_\omega = \frac{d\omega}{d\theta} = \frac{\partial \sigma}{\partial d} \frac{\partial d}{\partial t} + k \cdot \frac{\partial \vec{u}}{\partial t}$$

$$C_\theta = \frac{d\theta}{dt} = \frac{1}{k} \left( \frac{\partial \sigma}{\partial d} \frac{\partial d}{\partial m} + k \cdot \frac{\partial \vec{u}}{\partial m} \right)$$

in which $d$ is depth, $k$ is the wavenumber vector, $\vec{u}$ is the current velocity vector and $m$ is a coordinate orthogonal to the wave direction. These propagation speeds fully account for the depth and current effects on propagation in the linear theory of slowly-varying surface gravity waves.

The right hand side of Eq. (2) describes non-conservative processes which are:
1. Wind input $S_{in}$
2. Nonlinear resonant wave-wave interactions $S_{nl}$
3. Dissipation due to deep-water wave breaking $S_{db}$ (whitecapping)
4. Dissipation due to bottom friction $S_{bot}$
5. Depth-limited breaking dissipation in shallow water $S_{sh}$

The whitecapping