Single-Fluid 2D Magnetohydrodynamic Simulation of Solar Wind Structure in Comparison to Ulysses Observations

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Abstract: Two dimensional Magnetohydrodynamic (MHD) equations with and without the momentum addition respectively have been used to simulate the solar wind structure on the meridian plane. The results show that far away from the sun it isn't solar magnetic field that induces the concave solar wind speed. Instead, there may be the fast speed streamer driven by the momentum addition and an interface between high and low speed streamers. The interaction between high and low speed streamers causes the sharp division.

Key words: solar wind; solar magnetic; momentum addition
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0 Introduction

Solar wind plays an important role in the research of solar-terrestrial space environment. Slow solar wind is known to come from the top of the streamer. Coronal holes consist of magnetically open regions with low density and temperature in which fast solar wind is produced. Ulysses has observed that during solar minimum continuous fast solar wind is dominant at all latitudes except latitudes within 20° of the equator, generally in a range of 700 to 800 km/s, with a slight positive poleward speed gradient. And the division between fast and slow stream is very sharp[1]. Before Ulysses passed the high-altitude cusp regions, different kinds of latitudinal models of solar wind speed are put forward to describe three-dimensional structure of solar wind, which indicate that the solar wind speed obviously increases with latitude. Ulysses observations indicate these models can't explain the latitude distribution of solar wind. Therefore we should establish a new solar wind speed model. At first using the ideal magnetohydrodynamic (MHD) system as the governing equations, we try to simulate the latitude distribution of solar wind by adjusting the choice of the plasma parameters (such as T and β) at the bottom boundary or on the solar surface. But the results are unsatisfactory. Then we consider the effect of momentum addition without considering its mechanism which might be from Alfvén, sonic and magnetosonic waves. We adopt MHD equations with momentum addition as the governing equations to simulate the solar wind structure on the meridian plane, whose results agree well with Ulysses observations.

1 Governing Equations and Numerical Method

The model we use is for two dimensional MHD flow of a single-fluid, fully ionized plasma with momentum addition. The governing equations can be written as follows:

$$\frac{\partial p}{\partial t} = -\left(\frac{\partial (\rho v_z)}{\partial z} - \frac{\partial \left(\rho v_\theta \right)}{\partial \theta}\right) - \frac{2p_0}{r} - \frac{\rho v_\theta}{r \cot \theta}$$

(1)
\[
\frac{\partial v_r}{\partial t} = -v_r \frac{\partial v_r}{\partial r} - \frac{B_\theta}{\mu_0 r} \frac{\partial B_\theta}{\partial r} - \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{v_r}{r} \frac{\partial \rho}{\partial r}
\]
\[+ \frac{B_\theta}{\mu_0 r} \frac{\partial B_\theta}{\partial \theta} - \frac{B_\theta^2}{\mu_0 r} + \frac{v_\phi^2}{r^2} - \frac{GM_\odot}{r^3} + D(r, \theta)\] (2)

\[
\frac{\partial v_\phi}{\partial t} = -v_r \frac{\partial v_\phi}{\partial r} + \frac{B_r}{\mu_0 r} \frac{\partial B_r}{\partial r} - \frac{1}{\rho} \frac{\partial P}{\partial r} - \frac{v_\phi}{r} \frac{\partial \rho}{\partial r}
\]
\[- \frac{B_r}{\mu_0 r} \frac{\partial B_r}{\partial \phi} - \frac{v_r v_\phi}{r^2} + \frac{B_r B_\theta}{\mu_0 r}\] (3)

\[
\frac{\partial B_r}{\partial t} = \frac{\partial}{\partial r} \left( \frac{\partial B_\theta - \partial B_\phi}{r} \right) + \frac{1}{r} \left( v_r B_\theta - v_\phi B_\phi \right) \cot \theta
\]
\[(4)\]

\[
\frac{\partial B_\theta}{\partial t} = -\frac{\partial}{\partial r} \left( v_r B_\theta - v_\phi B_\phi \right) - \frac{1}{r} \left( v_r B_\theta - v_\phi B_\phi \right)
\]
\[(5)\]

\[
\frac{\partial P}{\partial t} = -\gamma P \frac{\partial P}{\partial r} - v_r \frac{\partial P}{\partial r} - \gamma P \frac{\partial v_r}{\partial r} - \frac{v_\phi}{r} \frac{\partial P}{\partial \phi}
\]
\[- \frac{\gamma P}{r} \left( 2v_r + v_\phi \cot \theta \right)\]
(6)

where \(r\) is heliocentric distance; \(\theta\) is the colatitude; \(\rho\) is the plasma density (\(\rho = n m_p\)); \(P\) is the isotropic thermal pressure; \(v_r\) and \(v_\phi\) are the flow velocities in the radial direction and meridional direction, respectively; \(B_r\) and \(B_\theta\) are magnetic field intensities in the radial direction and meridional direction, respectively. \(\mu_0\) is the magnetic permeability in the vacuum; \(\gamma\) is polytropic index; \(D\) is the momentum addition, formally given by

\[
D(r, \theta) = \frac{D_0 a^2}{(r - a)^2 + a^2} \cdot \{0.471 - 0.3 \arctan[20(7 - j)]\}
\]
(7)

where \(D_0\) is the strength of the momentum addition and its value is \(5 \times 10^{12} N/\text{g}^2\). The position of the maximum value of \(D(r, \theta)\) is \(a\) and fixed at 3.5 Rs (solar radius), and \(j\) is the index of the grid point in the direction of \(\theta\). Fig. 1 shows the momentum addition versus polar angle and solar radius. When we use the ideal MHD systems as the governing equations, \(D(r, \theta)\) is null.

The two step Lax-Wendroff scheme is used to numerically solve MHD equations above with the method of Flux-Corrected Transportation (FCT) algorithms adopted to stabilize the computation. The computation domain extends from the solar surface to 102 Rs in the radial direction and covers 183\(^\circ\) in azimuthal direction from \(-1.5^\circ\) to 181.5\(^\circ\). The grid spacings are \(\Delta r = 0.028 81 \times r\), and \(\Delta \theta = 3^\circ\), which gives a grid of 62 (meridional direction) \(\times 164\) (radial direction).

The governing equations are solved for two cases, one without momentum addition and one with momentum addition. They both use \(\gamma = 1.05\). There are four boundary conditions needed to specify. Since the flow is symmetric around and across the polar axis, symmetric boundaries are used along two poles. At the inner boundary, the flow is subsonic and sub-Alfvénic. According to the theory of projected normal characteristics, two of the six characteristics are negative. So we can only specify four independent variables. The other two are to be calculated from the compatibility equations that can be derived from the original set of MHD equations mentioned above. However it was pointed out by Steinolfson and Nakagawa that first order or second-order extrapolation often works as well using the more complex compatibility relations. Here, we chose \(v_r\) and \(v_\theta\) to be calculated by the second-order extrapolation. The other four variables are fixed throughout the computation. At the outer boundary, all variables are calculated by using the linear extrapolation because the flow is supersonic and super-Alfvénic. The other two boundary conditions are symmetric. This handling of the four boundaries is proved to be effective throughout the calculation.

Fig. 1 The spatial distribution of the momentum in dimensionless

In order to obtain the stable solar wind structure, we can use the method of time relaxation to solve above MHD equations. The plasma parameters on the solar surface are: density \(n_0 = 2.25 \times 10^{14} \text{ m}^{-3}\), temperature \(T_0 = 1.8 \times 10^6 \text{ K}\), the mag-