Some Remarks on the Cedent’s Retention Risk in Presence of an Annual Aggregate Deductible or Reinstatements

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1 Excess of Loss Reinsurance with Specific Clauses

Excess of loss reinsurance is widely used when it is the concern of the ceding company to cut the major claims individually. If $D$ is the retention of the Insurance Company, then the Reinsurer and the Cedent are liable for

$$ R_i = \max(0, X_i - D) $$

$$ C_i = X_i - R_i. $$

Often the reinsurer limits its liability to a level $L$. $[D, D+L]$ is called the layer. In this case (1.1) and (1.2) become

$$ R_i = \min(L, \max(0, X_i - D)) $$

$$ C_i = X_i - R_i, $$

where the subscript $i$ denotes the $i$th claim in the collective risk model. Let us mention that there may be more than one layer protecting the retention of the Cedent. But from now on we will assume, in order to make things easier, that there is only one layer. Obviously all the results remain true in case of more than one layer.

If there are $N$ claims during the year, the aggregate claims distribution of the Reinsurer and the Cedent are

$$ S_R = R_1 + \ldots + R_N $$

$$ S_C = C_1 + \ldots + C_N. $$

Excess of loss reinsurance is an individual reinsurance in the sense that the payments of the Reinsurer depend on the individual claims amounts. There is a theoretical result (cf. Denuit and Vermandele (1998)) that says that among all individual reinsurances with the same expected reinsurance loss and the same reinsurance premium, excess of loss reinsurance is optimal in the sense that it is not possible to find any other type of reinsurance with a retained risk less risky in the sense of the stop loss order. This theoretical result explains why excess of loss reinsurance is so common on the reinsurance market. However it is generally combined with some specific clauses, as we will see below.
1.1 Typical Clauses for Excess of Loss Reinsurance

In this section we will review some typical clauses that may be found on the market, in combination with excess of loss reinsurance. We will show how to price these clauses and their effect on the retained risk of the Ceding company. But first let us define some quantities that are interesting for examining these clauses.

1.1.1 Definition of Quantities

Let us define $P$ the insurance premium and $P_{Re}$ the reinsurance premium. Then $P_{Ced}$ is the "net of reinsurance" insurance risk premium, i.e.

$$P_{Ced} = P - P_{Re}.$$ 

The aggregate claims distribution of the Reinsurer is a function of $R$ and $N$:

$$S_{Re} = f(R, N).$$

The aggregate claims distribution of the Cedent is a function of $R$, $C$ and $N$:

$$S_{Ced} = f(R, C, N).$$

Further in this paper we will show how to calculate the reinsurance premium. Sometimes this reinsurance premium will be a random variable:

$$P_{Re} = p_{Re}^{det} + p_{Re}^{rand},$$

where $P_{Re}^{det}$ is the part of the reinsurance premium that is deterministic whereas $P_{Re}^{rand}$ is the random part of $P_{Re}$. $P_{Re}^{det}$ is the part of the reinsurance premium that will be paid for sure to the reinsurer. $P_{Re}^{rand}$ is the part of the reinsurance premium that will be paid in accordance with the observed claims hitting the Reinsurer. It is therefore a random variable.

Furthermore we will study the ruin probability of the ceding company. In order to apply the algorithms related to ruin probability, it is necessary to have a Cedent premium $P_{Ced}$ that is not random:

$$P_{Ced} = p_{Ced}. $$

This is the reason why we will refer to a Cedent claims distribution $S_{Ced}$ that is constructed in such a way that $P_{Ced}$ is deterministic. This means that the randomness of the $P_{Ced}$, due to $P_{Re}$, will be transferred to $S_{Ced}$:

$$S_{Ced} = f(R, C, N) + P_{Re}^{rand}. $$

The reinsurance premium $P_{Re}$ (or more precisely $P_{Re}^{det}$) will be calculated according to a premium principle $H$, which we will discuss further on in this paper. Let us now review different kinds of clauses.