Anomalous Moment of the \( \mu \)-Meson
for Different Models of Breakdown of quantum Electrodynamics.

B. De Tolle

Istituto di Fisica dell'Università - Roma
Istituto Nazionale di Fisica Nucleare - Sezione di Roma

(ricceto il 2 Marzo 1960)

In a paper published some time ago, Berestetskij et al. (1), calculated the correction of the \( \mu \)-meson anomalous magnetic moment due to a possible modification of the photon propagator in case of a break-down of quantum electrodynamics at small distances. Numerically, for the \( \mu \)-meson, the correction is of the order of \( \alpha (\alpha = 1/137) \) and therefore it adds directly to Schwinger’s \( \alpha/2\pi \) anomalous contribution (for the electron, because of its small mass, the deviation would be much smaller and unobservable).

The model that has been used consists in substituting to the photonic propagator \( 1/k^2 \) the modified propagator

\[
\frac{1}{k^2} \frac{A^2}{k^2 + A^2} \frac{1}{k^2 + A^2},
\]

where \( A \) is the cut-off \( (2) \) \( (\hbar = e = 1) \), and it leads to the following results:

\[
\frac{\delta \mu}{\mu} = \frac{\alpha}{2\pi} (1 - \delta F),
\]

where \( \mu = e/2m_\mu \) and

\[
\delta F = 2 \int_0^1 \frac{(1 - x)x^2}{x^2 + \eta x + \eta} \, dx;
\]

\[
\delta F \approx \frac{2m_\mu^2}{3A^2}, \quad \text{for } \eta \gg 1.
\]


In this note we evaluate what would be the correction to the muon magnetic moment from a possible modification either of the muon propagator or of the electromagnetic vertex. We first calculate the correction from a possible modification of the muon propagator. We substitute the modified propagator

$$\frac{1}{(p + k)^2 + m^2} - \frac{A^2}{(p + k)^2 + m^2 + A^2'},$$

where $A'$, represents the photon 4-momentum. Putting in the final expression $p = p'$, we find the following correction to the anomalous moment

$$\frac{\Delta \mu}{\mu} = \frac{2}{3\pi} \left(1 - \Delta F'\right),$$

where

$$\Delta F' = \frac{1}{4\eta^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{x(1 - x) y^2 z}{x + (1 - z) y (1 - y z)} = \frac{2}{3} \left[ \frac{1}{2} - \eta + \frac{1 + \eta}{\eta} \ln (1 + \eta) \right. \left. + 2 \eta^2 \ln \eta \right],$$

and

$$\Delta F' \approx \frac{2}{3 \eta} \left( \ln \eta + \frac{1}{3} \right),$$

for $\eta \gg 1$.

We have assumed a possible vertex modification to depend for simplicity only from the virtual photon momentum $k$, according to the substitution

$$\gamma^\mu \rightarrow \frac{A^2}{k + A^2 \gamma^\mu}.$$

Only two vertices must be modified. The vertex with external photon is not modified in the limit $q = 0$ ($q = p' - p$). We obtain

$$\frac{\Delta \mu}{\mu} = \frac{2}{3\pi} \left(1 - \Delta F''\right),$$

where

$$\Delta F'' = \frac{1}{4 \eta^2} \int_0^1 dx \int_0^1 dy \int_0^1 dz \frac{x^2 y^2 (1 - x) (1 - x y)}{(x^2 y^2 + \eta (1 - x))^2};$$

$$\Delta F'' = \frac{1}{1 - \frac{4}{\eta}} \left[ \frac{\eta(1 - \frac{4}{\eta})}{2} \ln \eta - \frac{\eta^2 - 6 \eta + 4}{2 \eta \sqrt{1 - \frac{4}{\eta}}} \ln \left(1 + \sqrt{1 - \frac{4}{\eta}}\right) \right]$$