Validity of Q.E.D. in \( \mu \) Pair Production.

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Recently it was noticed (1) that the actual technical improvements make possible, using colliding beams, the realization of high energy experiments. Particularly all these experiments concern the interaction of electrons with electrons and positrons. Moreover, while electron-electron scattering happens only with the exchange of a photon (to \( \epsilon^2 \) order), electron-positron interaction can take place also via the annihilation and subsequent recreation of the electron positron pair. If the acting energy is sufficiently high (higher than a certain threshold) near the primitive process, the production of different particles, in number and conditions fixed by the various selection rules will become possible. Beside, calling \( p_x \) and \( p \) the momenta of the positron and electron, the momentum transfer in the annihilation of the pair is

\[
q^2 = (p_x + p)^2
\]

and in the C.M.S. \( q^2 = -4\epsilon^2 \), independent of the angle and far away from the mass-shell. So, we understand that this form of the interaction \( e^+e^- \) is very much favourable for an inquiry on the validity at short distances of the fundamental conceptions of the actual quantum field theory. In fact, as proposed by many authors, electron-positron annihilation can give particularized information on the limits of validity of the Q.E.D. (2,3) and on the structure of the elementary particles.

For example CARIBBO and GATTO (4) have shown the possibility of measuring directly the photon-pion vertex (and the correspondent form factors) with processes of the type:

\[
e^+ + e^- \rightarrow n \text{ pions}.
\]

The object of this short note is to study the production of \( \mu \mu \) pairs in the \( e^+e^- \) annihilation.

The presence of the $\mu$-mesons should reveal favourable for an inquiry at small distances, because their Compton wave-length, less than that of the electron, is just of the linear dimensions of the regions that we wish to explore.

We assume that the $\mu$-meson is not punctiform and that its interaction with the electromagnetic field is given by:

$$ F_\mu = \gamma_\mu F'_1(q^2) + \sigma_{\mu\nu} q_\nu F'_2(q^2) \quad \sigma_{\mu\nu} = \frac{\gamma'_\mu \gamma'_\nu - \gamma'_\nu \gamma'_\mu}{2i}. $$

If the $\mu$-meson doesn't allow other interactions different from the electromagnetic one, such a form of its vertex takes into account of a violation the Q.E.D. In the limits of a non-relativistic treatment we can identify $F'_1$ with Fourier's transform of the charge distribution of the $\mu$-meson. The second term, containing $F'_2(q^2)$ means a different possibility of describing an alteration of the local electrodynamics and with it we can refer to an eventual magnetic structure of the $\mu$-meson.

For the conventional electrodynamics $F_\mu = \gamma_\mu$ that is $F'_1 = 1$, $F'_2 = 0$ (or more exactly the term containing $F'_2$ is < 0 for quantities of the order $z = 1/137$ and contributes only to the radiative corrections). Together with the condition to vanish for large $q^2$, this requires that the two form factors satisfy the boundary conditions:

$$ F'_1(0) = 1 \quad F'_1(\infty) = 0, $$

$$ F'_2(0) = 0 \quad F'_2(\infty) = 0. $$

In the approximation to order $\varepsilon^2$, the process is described by a diagram only and the cross-section gives easily the expression valid in every reference frame.

$$ (2) \quad d\sigma = \frac{\alpha^2}{4m^4(1+\varepsilon)^2\sqrt{x^2-1}} \beta_\varepsilon^2 \varepsilon \frac{dM'}{d^2p_+} \cdot \left\{ 2F^2_1(q^2)[(1+x)(M^2 + m^2) - M^2(\mu^2 + \lambda^2)] \right\} \cdot \{ 4m^2E_1(q^2)E_2(q^2)(1+x)(2-x) + m^2F^2_2(q^2)(1+x)(3+x+2\mu\lambda)\}, $$

where

$$ E = \varepsilon_+ + \varepsilon_-, \quad p = p_- + p_+ $$

$\theta'_-$ is the angle between $p$ and $p'_-$; $q^2$ is the transfer impulse; $\mu$, $\lambda$, $k$ are invariants defined by:

$$ \cdot m^2 k = p_+ p_-, $$

$$ \cdot m \lambda = p_+ p'_- = p_- p'_-, $$

$$ \cdot m \mu = p_+ p'_+ = p_- p'_+, $$

(*) Note: A form of the electromagnetic vertex equal to (1) has been used by Avakov (*) for an analysis of the validity of the Q.E.D. in the scattering $e^-\text{He}$.  