XI. The Relation between Mathematics and Physics

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The physicist, in his study of natural phenomena, has two methods of making progress:
(1) the method of experiment and observation, and (2) the method of mathematical reasoning. The former is just the collection of selected data; the latter enables one to infer results about experiments that have not been performed. There is no logical reason why the second method should be possible at all, but one has found in practice that it does work and meets with remarkable success. This must be ascribed to some mathematical quality in Nature, a quality which the casual observer of Nature would not suspect, but which nevertheless plays an important role in Nature's scheme.

One might describe the mathematical quality in Nature by saying that the universe is so constituted that mathematics is a useful tool in its description. However, recent advances in physical science show that this statement of the case is too trivial. The connection between mathematics and the description of the universe goes far deeper than this, and one can get an appreciation of it only from a thorough examination of the various factors that make it up. The main aim of my talk to you will be to give you such an appreciation. I propose to deal with how the physicist's views on this subject have been gradually modified by the succession of recent developments in physics, and then I would like to make a little speculation about the future.

Let us take as our starting-point that scheme of physical science which was generally accepted in the last century – the mechanistic scheme. This considers the whole universe to be a dynamical system (of course an extremely complicated dynamical
system), subject to laws of motion which are essentially of the Newtonian type. The role of mathematics in this scheme is to represent the laws of motion by equations, and to obtain solutions of the equations referring to observed conditions.

The dominating idea in this application of mathematics to physics is that the equations representing the laws of motion should be of a simple form. The whole success of the scheme is due to the fact that equations of simple form do seem to work. The physicist is thus provided with a principle of simplicity, which he can use as an instrument of research. If he obtains, from some rough experiments, data which fit in roughly with certain simple equations, he infers that if he performed the experiments more accurately he would obtain data fitting in more accurately with the equations. The method is much restricted, however, since the principle of simplicity applies only to fundamental laws of motion, not to natural phenomena in general. For example, rough experiments about the relation between the pressure and volume of a gas at a fixed temperature give results fitting in with a law of inverse proportionality, but it would be wrong to infer that more accurate experiments would confirm this law with greater accuracy, as one is here dealing with a phenomenon which is not connected in any very direct way with the fundamental laws of motion.

The discovery of the theory of relativity made it necessary to modify the principle of simplicity. Presumably one of the fundamental laws of motion is the law of gravitation which, according to Newton, is represented by a very simple equation, but, according to Einstein, needs the development of an elaborate technique before its equation can even be written down. It is true that, from the standpoint of higher mathematics, one can give reasons in favour of the view that Einstein’s law of gravitation is actually simpler than Newton’s, but this involves assigning a rather subtle meaning to simplicity, which largely spoils the practical value of the principle of simplicity as an instrument of research into the foundations of physics.

What makes the theory of relativity so acceptable to physicists in spite of its going against the principle of simplicity is its great mathematical beauty. This is a quality which cannot be defined, any more than beauty in art can be defined, but which people who study mathematics usually have no difficulty in appreciating. The theory of relativity introduced mathematical beauty to an unprecedented extent into the description of Nature. The restricted theory changed our ideas of space and time in a way that may be summarised by stating that the group of transformations to which the space-time continuum is subject must be changed from the Galilean group to the Lorentz group.