Effects of the Biquadratic-Exchange Interaction in Heisenberg Ferromagnet.

M. Tiwari and R. N. Srivastava

Applied Physics Section, Institute of Technology
Banaras Hindu University - Varanasi 221005

Summary. — The effects of the biquadratic-exchange interaction on magnetization and susceptibility for spin-1 systems are investigated. The simple Green's function technique using Callen-type decoupling and RPA decoupling has been utilized to obtain the spin wave energies. Spin wave dispersion has been discussed. Limitations of decoupling have been indicated.

1. — Introduction.

The existence of the biquadratic-exchange interaction has been pointed out by Harris and Owen (1) and Anderson (2). Liu et al. (3) postulated an additional term in free energy in the form of cosine of the twice the angle between the magnetization of two sublattices. Kittel (4) mathematically obtained an expression for free energy which includes a term \((M_A \cdot M_B)^2\) and thus physically justified the postulate of Lie. Allen and Betts (5) mentioned that the permutation operator for spin 1 is not \(P_{ij} = 2S_i S_j + \frac{1}{2}\) as for spin \(\frac{1}{2}\), but instead \(P_{ij} = S_i S_j + (S_i \cdot S_j)^2 - 1\), and thus gave a possible origin of biquadratic ex-

---

change. When the balance between the exchange and elastic forces is established, it appears that the spin Hamiltonian having the form \((S_i \cdot S_j)^2\) may arise and this term has been included in writing the expression for the Hamiltonian in the system under consideration. Brown (6) has discussed the biquadratic exchange for spin 1 by using a molecular-field approximation. For the spin \((S = \frac{1}{2})\) ferromagnet, the biquadratic interaction is reduced to the bilinear one by simple identity. For spin 1 and higher spins there is no simple identity to reduce the biquadratic term to the bilinear one and, therefore, the biquadratic term with an arbitrary coefficient remains. At the same time, the bilinear exchange interaction does not provide a complete description of magnetic properties of the magnetic compounds and, therefore, we need to include the biquadratic-exchange interaction.

The Hamiltonian for such a system is given by

\[
\mathcal{H} = - \mu H \sum_i S_i^z - 2J \sum_{\langle i,j \rangle} S_i \cdot S_j - 2\alpha J \sum_{\langle i,j \rangle} (S_i \cdot S_j)^2,
\]

where \(\mu\) is the magnetic moment, \(H\) is the external field, \(S_i\) and \(S_j\) are the neighbouring spins of a regular lattice. \(J\) is the exchange integral constant and \(\alpha\) (alpha) is the biquadratic parameter. The summation extends over all \(i\)'s and \(j\)'s. \(\alpha\) varies from 0 to 1 and it measures the strength of biquadratic interaction. This model has been studied by the authors (7). They have used RPA decoupling, which does not yield very accurate results. Since the effect of the biquadratic term is smaller than that of the bilinear one, the biquadratic term should be decoupled more accurately and this can be achieved by Callen-type decoupling. The method which we describe here is based on the Green's function technique followed by Callen (8). We have shown calculations for magnetization below the Curie temperature, at the Curie temperature and susceptibility above the Curie temperature. The variation of the Curie temperature with different \(\alpha\)'s for s.c., b.c.c. and f.c.c. crystal lattices has been studied. We have obtained the variation of magnetization and the susceptibility with \(KT/J\) for different \(\alpha\)'s. The results so obtained have been compared with the experimental curves and also with the theoretical curves obtained by MFA theory. Spin wave dispersion relations have been obtained for low and high energies at room temperature, near the Curie temperature and above the Curie temperature for different \(\alpha\)'s. The theoretical curves have been compared with the experimental one for iron. Finally, we have discussed the limitations of our decoupling used in the description of the above model.