Effects of the Biquadratic-Exchange Interaction in Heisenberg Ferromagnet.

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Summary. — The effects of the biquadratic-exchange interaction on magnetization and susceptibility for spin-1 systems are investigated. The simple Green's function technique using Callen-type decoupling and RPA decoupling has been utilized to obtain the spin wave energies. Spin wave dispersion has been discussed. Limitations of decoupling have been indicated.

1. — Introduction.

The existence of the biquadratic-exchange interaction has been pointed out by Harris and Owen (1) and Anderson (2). Liu et al. (2) postulated an additional term in free energy in the form of cosine of the twice the angle between the magnetization of two sublattices. Kittel (4) mathematically obtained an expression for free energy which includes a term \((M_A \cdot M_B)^2\) and thus physically justified the postulate of Lie. Allen and Betts (5) mentioned that the permutation operator for spin 1 is not \(P_{ij} = 2S_iS_j + \frac{1}{2}\) as for spin \(\frac{1}{2}\), but instead \(P_{ij} = S_iS_j + (S_i \cdot S_j)^2 - 1\), and thus gave a possible origin of biquadratic ex-

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change. When the balance between the exchange and elastic forces is estab-
lished, it appears that the spin Hamiltonian having the form \((S_1 \cdot S_2)^2\) may
arise and this term has been included in writing the expression for the Ha-
iltonian in the system under consideration. BROWN (6) has discussed the bi-
quadratic exchange for spin 1 by using a molecular-field approximation. For
the spin \(S = \frac{1}{2}\) ferromagnet, the biquadratic interaction is reduced to the
bilinear one by simple identity. For spin 1 and higher spins there is no
simple identity to reduce the biquadratic term to the bilinear one and, there-
fore, the biquadratic term with an arbitrary coefficient remains. At the same
time, the bilinear exchange interaction does not provide a complete description
of magnetic properties of the magnetic compounds and, therefore, we need to
include the biquadratic-exchange interaction.

The Hamiltonian for such a system is given by

\[
H = - \mu H \sum_i S_i^2 - 2J \sum_{\langle ij \rangle} S_i \cdot S_j - 2\alpha J \sum_{\langle ij \rangle} (S_i \cdot S_j)^2,
\]

where \(\mu\) is the magnetic moment, \(H\) is the external field. \(S_i\) and \(S_j\) are the
neighbouring spins of a regular lattice. \(J\) is the exchange integral constant
and \(\alpha\) (alpha) is the biquadratic parameter. The summation extends over
all \(i\)'s and \(j\)'s. \(\alpha\) varies from 0 to 1 and it measures the strength of biquadratic
interaction. This model has been studied by the authors (7). They have used
RPA decoupling, which does not yield very accurate results. Since the effect of
the biquadratic term is smaller than that of the bilinear one, the biquadratic term
should be decoupled more accurately and this can be achieved by Callen-type
decoupling. The method which we describe here is based on the Green’s func-
tion technique followed by CALLEN (8). We have shown calculations for magnet-
ization below the Curie temperature, at the Curie temperature and susceptibility
above the Curie temperature. The variation of the Curie temperature with
different \(\alpha\)'s for s.c., b.c.c. and f.c.c. crystal lattices has been studied. We
have obtained the variation of magnetization and the susceptibility with
\(K T / J\) for different \(\alpha\)'s. The results so obtained have been compared with the
experimental curves and also with the theoretical curves obtained by MFA
theory. Spin wave dispersion relations have been obtained for low and high
energies at room temperature, near the Curie temperature and above the Curie
temperature for different \(\alpha\)'s. The theoretical curves have been compared with
the experimental one for iron. Finally, we have discussed the limitations of
our decoupling used in the description of the above model.

(7) J. ADLER, J. OITMAA and A. M. STEWART: Physica (B+C) (The Hague), 86-88,
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