Calculation of Some Solutions of the Bobylev-Krook-Wu and Tjon-Wu Equations.

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Summary. — Two fully nonlinear Boltzmann equations for infinite spatially uniform gases consisting of a single species of particle are considered. The models are the one due to Bobylev, Krook and Wu and the one due to Tjon and Wu. These are shown to be related by the Abel transform and an existence uniqueness theorem is presented. Then a procedure for the numerical calculations of solutions—based upon the method of Barnsley and Cornille—is given and a device for speeding up its convergence is described. Finally, several numerical examples are discussed. Some enhancement is shown to occur, but not enough to be physically important.

1. - Introduction.

In this paper we present some theoretical and numerical results for two model nonlinear Boltzmann equations. We consider the equation for Maxwell molecules which was studied by Bobylev (1) and by Krook and Wu (2) (the BKW equation) and also the related model of Tjon and Wu (3) (the TW equa-

tion). It is one of our aims to fill out some previously reported observations \(^4,^5\) and to present a new existence theorem.

Solution of the nonlinear Boltzmann equation generally requires the use of approximations of unknown accuracy. Indeed one is often not even assured of the existence and uniqueness of the sought after solutions. Hence the discovery by Bobylev, and independently by Krook and Wu, of an explicit formula for a one-parameter family of solutions to the BKW equation has stimulated much research interest. This family of solutions is referred to as the BKW mode and also as the similarity solution. It is related—through quite complicated changes of variable—to certain explicit solutions for spatially nonuniform systems \(^6\).

The TW equation is a simplified form of the BKW equation and it was introduced to provide a model which was more amenable to numerical studies. It was used to examine the conjecture of Krook and Wu that any solution of the BKW equation initially relaxes to the similarity solution, which it then follows to equilibrium. In its broadest interpretation this conjecture was shown to be wrong by Barnsley and Cornille \(^7,^8\) and also by Hauge \(^9\). More recently Hendriks et al. \(^10\) have put forward a modified version of the conjecture. However, the TW equation is of interest for other reasons. First, it is a model Boltzmann equation in its own right and hence the behavior of its solutions can be expected to display qualitative features which are equally well true for the BKW equation. Second, although it was originally conceived as being related to the BKW equation through two integral transforms, one with an apparently complicated kernel, it has since been observed by Barnsley and Turchetti that the direct connection between the BKW and TW equations is provided by the Abel transform. The derivation of this result is given in sect. 4. An important consequence of it is that any proposition concerning the nature of solutions of the TW equation can be directly transformed back to provide a corresponding statement for the BKW model. For example, one readily deduces expansion \((7.10)\) for solutions of the BKW equation from expansion \((6.8)\) for solutions of the TW equation.

The formal series expansions, in terms of Laguerre polynomials, for the general solutions of the TW and the BKW equations, have been given by


