Summary. — The flux of representative points for a 3-3 scattering is analysed across a sphere of radius $q$ in the 6-dimensional space of relative co-ordinates. It is shown that, despite the «anomalous» behaviour of the double scattering, all terms are finite. The leading $O(q)$ and $O(q^4)$ terms are computed for the full stationary wave; they turn out to depend only on on-mass-shell two-body $S$ matrices and (correctly) cancel by virtue of the two-body optical theorem. Some of the $O(1)$ terms are shown explicitly; they refer to those regions where some pair is still interacting, but depend only on Wigner's time delay.

1. — Introduction.

As is well known, the three-body scattering problem with pairwise potentials presents peculiar complications arising from the fact that two particles may interact while being very far from the third particle. Whether or not there exist bound states, of the three or of any pair, the above entails a very complicated asymptotic motion in configuration space. This apparently motivated Faddeev (1) to study the problem in momentum space; after the momentum-space singularities of the three-body $T$-matrix were elucidated, it was possible (2-5) to deduce the asymptotic form of the $x$-space wave function.

The asymptotic form of the scattered wave cannot be so simple as, say, in the corresponding two-body problem. The two-body problem really reduces, after taking off the centre-of-mass motion, to one-body scattering off a potential localized around the origin. If the potential decreases fast enough at infinity, the incoming plane wave is scattered into a spherical wave that falls off like the Green’s function of $R_3$. The three-body problem, after taking off the centre of mass, reduces to a 6-dimensional problem in the relative co-ordinates. In this space, however, the sum of the pair potentials is not zero outside any finite sphere. The incoming plane wave is scattered into a wave that cannot behave like the Green’s function of $R_6$.

There are, in fact, two reasons why this must be so. Firstly, on any spherical surface (no matter how large) there are portions in which a pair is still interacting and whose wave function is not developing as in free motion. Secondly, the incoming plane wave hitting infinitely extended "obstacles" (like, say, the potential $V_{12}$) may scatter into a wave much larger that would be radiated by a point source (Green’s function). This latter argument is made fully convincing by thinking of the analogy (6) with the scattering of a plane wave off an infinite wedge in $R_3$, as against scattering off a small object. The infinite wedge gives rise to geometrical reflections plus diffraction, and the exact wave must deal correctly with both also providing for the matching (7).

In the three-body problem the waves analogous to the geometrical-optics solution have been shown (2-5) to be those arising from single scattering (one particle goes straight through, unconnected) and those arising from double scattering with intermediate propagation on the mass shell ("anomalous" double scattering). Letting aside the unconnected terms, whose structure is quite obvious, the "anomalous" double scatterings appear to give rise to bothersome infinities, so that authors have tried to find out ways to subtract their contribution so as to be left with a "truly three-body amplitude" of finite cross-section. These attempts have largely been unsuccessful because no "clean" way of performing the subtraction was found, the problem being of not to subtract too much (2) and of assessing the magnitude of the interference of asymptotic waves (5,6).

We believe an answer can be given if one is able to handle appropriately the co-ordinate space wave function. The known asymptotic behaviour refers to a stationary solution corresponding to an incoming plane wave. Hence the flux through any closed surface must be identically zero. If the surface is taken to be a large spherical surface in $R_6$ of radius $q$ and the flux, in this kind of space, is expanded as $q \to \infty$, the asymptotic expansion must vanish.