A quark model based on QCD scale anomaly

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Abstract. We consider a quark model based on QCD scale anomaly in which the quarks move in the field of an effective glueball field. Hadrons correspond to solitonic bags of higher energy density in a nonperturbative sea of condensed gluons. We calculate the static properties of nucleon in this model and find that the nucleon mass is far too large (2.4–4 GeV) and the proton charge radius (0.37–0.54 fm) is low. The proton gyromagnetic ratio and $g_A/|g_V|$ are in reasonable agreement with the experimental numbers.

Keywords. Quarks; scale anomaly; glueball; soliton; nucleon; static properties; quantum chromodynamics.

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1. Introduction

At present, it is not possible to derive the low energy properties of hadrons from the first principles of QCD. This is because the effective coupling constant of QCD is large at low energies and perturbative techniques cannot be applied in this regime. Hence, one has to rely on phenomenological models in the low energy region. The various characteristic features of QCD provide the basis for the construction of such models. For instance, in the MIT bag model, confinement is assumed and the quarks are restricted to a bag which is identified with a hadron (Chodos et al 1974). Starting from some general arguments based on the ‘diaelectric’ nature of QCD vacuum, one can construct a class of solitonic bag models in which the bag formation is due to the interaction of quarks with an effective gluon condensate scalar field (see for instance Lee 1981). Here the quark-scalar field interaction is of the familiar Yukawa type but the potential corresponding to the self-interaction of the scalar field is arbitrary subject to some general restrictions. The MIT and SLAC bag models are obtained in some limiting situations in this class of models. In the Skyrme model, baryons are the topological solitons of a chiral invariant, nonlinear mesonic field theory (for a review see Zahed and Brown 1986). There are also, other approaches like hybrid models in which both the quarks and mesonic degrees of freedom are present with chiral invariant couplings (Banerjee 1987). Each of the models has its own advantages and limitations and none of them can claim to have a distinct advantage over the others.

One of the distinguishing features of QCD is its anomalous behaviour under scale transformations (Nielsen 1977; Collins et al 1977). Indeed the vacuum expectation value of the scale anomaly sets the confinement scale which is one of the two fundamental scales of QCD, the other being the chiral symmetry-breaking scale. The
QCD scale anomaly can be incorporated in an effective Lagrangian framework through a 'glueball' field (Schechter 1980; Solomone et al 1981). It has been shown that a bag can be automatically formed in the Skyrme model modified to possess the correct QCD scaling behaviour (Gomm et al 1986; Jain et al 1987). In this paper we chose to work with a quark model based on the scale anomaly. The model is economical in the sense that we have only the quark and glueball degrees of freedom with the simplest possible interactions. Using suitable approximations, we find that there are nontopological solitonic solutions in this model, which we identify with hadrons. In other words, hadrons are bubbles of higher energy density in a nonperturbative sea of condensed gluons. Our model is similar in spirit to the solitonic bag models mentioned earlier but the potential corresponding to the scalar self-interactions is completely fixed by the scale anomaly. For reasonable ranges of values for the glueball mass and coupling constant associated with the quark-glueball interactions which are the only free parameters in this model, we find that the nucleon mass comes out too large (2-4-4 GeV) and the proton charge radius (0.37-0.54 fm) is smaller than the experimental value (≈ 0.8 fm). However the dimensionless parameters fare better. $g_A/g_V$ is in the range (1.1-1.26) compared with the experimental value of 1.23 and the proton gyromagnetic ratio $\mu_p$ is (1.97-3.05), the measured value being 2.79. This is to be compared with the results of the modified Skyrme model where the electric and magnetic charge radii are predicted to be within 10-15% of the experimentally measured quantities after fitting the $N$ and $\Lambda$ masses (at the cost of pushing down the value of $F_\pi$) and $g_A/g_V$ is very low (≈ 0.60).

The paper is planned as follows: in §2 we set up the effective Lagrangian and the equations of motion. We give details of the approximation scheme we employ to obtain the soliton energy in §3 and discuss the soliton solutions in §4. In §5, we calculate the static properties of the nucleon in this model and present the numerical results. We make a few concluding remarks in the end.

2. A quark-glueball interaction model

In QCD with massless quarks the divergence of the scale current or dilation current $J_\mu^D$ is zero from naive calculations. A more careful calculation yields the scale anomaly equation (Nielson 1977; Collins et al 1977):

$$\partial_\mu J_\mu^D = \theta^\mu = \frac{\beta(g)}{g} \text{Tr} (F_{\mu\nu}F^{\mu\nu}) = H.$$  \hspace{1cm} (1)

Here $\theta^\mu$ is the trace of the energy momentum tensor. From QCD sum rules (Shifman et al 1979)

$$\langle \partial_\mu J_\mu^D \rangle \simeq (0.34 \text{ GeV})^4.$$  \hspace{1cm} (2)

First, let us consider a simple effective Lagrangian for QCD without matter fields. We assume that the degrees of freedom reduce to that of the single order parameter field $H$ defined in (1). Then it can be shown (Schechter 1980; Solomone et al 1981) that the unique Lagrangian (up to two derivatives) which satisfies the scale anomaly equation is:

$$\mathcal{L}_H = \frac{1}{2} b^2 H^{-3/2}(\partial_\mu H)^2 - \frac{1}{2} H \ln (H/\Lambda^4),$$  \hspace{1cm} (3)