Modeling of individual coherent structures in wall region of a turbulent boundary layer*

ZHOU Heng(周 恒), LU Changgen(陆昌根) and LUO Jisheng(罗纪生)
(Department of Mechanics, Tianjin University, Tianjin 300072, China)

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Abstract  Models for individual coherent structures in the wall region of a turbulent boundary layer are proposed. Method of numerical simulations is used to follow the evolution of the structures. It is found that the proposed model does bear many features of coherent structures found in experiments.

Keywords: turbulent boundary layer, modeling, coherent structure.

The investigation of coherent structures in the wall region of a turbulent boundary layer has greatly enriched our knowledge of turbulence. However, most investigations, such as the detection, identification and description, were of kinematic nature. In recent years, the group in Tianjin University has done some work on dynamic modeling of coherent structures[1-3], in which, models of coherent structures were proposed, which were solutions of the disturbance equations of a certain basic flow. Also, the proposed models were found to be useful, at least for the computation of the transport problem of passive quantities[4].

However, the proposed model suffers from the shortcomings that it was based on the idea of instability waves which is a certain kind of wave train, while the experimental observations suggest that individual coherent structures do not have strong interactions with other ones. In this paper, a further step is taken to improve the model proposed, which agrees better with what were observed in experiments.

1 The numerical method

The basic equation is the Navier-Stokes equation:
\[
\frac{\partial u}{\partial t} + (u \cdot \nabla) u = - \nabla p + \frac{1}{R} \nabla^2 u, \quad \nabla \cdot u = 0,
\]
where \( u \) is the velocity, \( p \) the pressure, \( R \) the Reynolds number, \( \nabla \) the gradient operator, \( \nabla^2 \) the Laplacian.

But for our purpose, the disturbance equation derived from the N-S equation is more appropriate. We decompose the velocity and pressure as
\[
u = U + \tilde{u}, \quad p = P + \tilde{p},
\]
where \( U \) is the velocity of the basic flow, \( \tilde{u} \) the disturbance velocity, \( P \) the pressure of the basic flow, \( \tilde{p} \) the disturbance pressure. Then the equations for the disturbance are

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The basic flow is supposed to be known, which will be described later. Since the region we are concerned is very close to the wall, about within 100 viscous length from the wall, the small scale turbulence is very weak there. This assertion is supported by several reasons, but the most direct evidence can be found in a figure of ref. [5], obtained by direct numerical simulations, in which the velocity field in a normal cross-section at the location where a coherent structure existed was shown. Apart from the large eddies of the size of coherent structures, there is no obvious small scale motion. Therefore, in our calculations, quasi-laminar assumption is adopted.

The basic flow is obtained by solving the N-S equation until the solution becomes stationary, subject to the boundary conditions that it joins smoothly the mean turbulent profile at \( y^* = 100 \), where the plus sign implies that \( y \) is measured in terms of viscous length scale, and since the thickness of the boundary layer is varying in the streamwise direction, if we limit ourselves in a rectangular domain for the computation, \( y^* \) is not strictly constant at the upper boundary. The in-flow and out-flow boundary condition is given by a profile consisting of a part of Blasius profile, appropriately compressed in \( y \) direction, joining smoothly with the local mean turbulent profile at the upper boundary. Such a profile has been used in refs. [1, 2] for a local analysis, and its reasonableness has been discussed elsewhere and will not be repeated here.

The integration of eq. (1) or (2) is carried out as follows: first, we Fourier decompose every variable in spanwise, i.e. \( z \) direction as

\[
\hat{\phi}(x, y, z, t) = \sum_{n=-N/2}^{N/2} \phi_n(x, y, t) e^{-im\beta z},
\]

where \( \phi \) is a representative flow quantity, \( N \) the number of collocation points in \( z \) direction. The non-linear term in eq. (1) is Fourier decomposed separately as

\[
(u \cdot \nabla) u = \sum_{n=-N/2}^{N/2} \mathcal{F}_n[(u \cdot \nabla) u] e^{-im\beta z}.
\]

Then eq. (1) will be reduced to a system of 2-D equations as

\[
\frac{\partial \mathbf{u}_m}{\partial t} + \mathcal{F}_m = -\nabla \cdot \mathbf{p}_m + \frac{1}{R} \nabla^2 \mathbf{u}_m,
\]

\[
\nabla \cdot \mathbf{u}_m = 0, \quad m = -\frac{N}{2}, \ldots, \frac{N}{2} - 1,
\]

where \( \nabla_m = \left[ \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, -im\beta \right]^T \), \( \nabla^2_m = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} - m^2 \beta^2 \).

For the integration of eq. (6), we use mixed explicit-implicit time splitting technique, namely, three steps are taken as

\[
\frac{\mathbf{u}_m - \sum_{q=0}^{2} \alpha_q \mathbf{u}_m^{n-q}}{\Delta t} = -\sum_{q=0}^{3} \beta_q \mathcal{F}_m,
\]

\[
\frac{\mathbf{u}_m^{n+1} - \mathbf{u}_m'}{\Delta t} = -\nabla \cdot \mathbf{p}_{m}^{n+1},
\]

\[
\frac{\mathbf{p}_m^{n+1} - \mathbf{p}_m'}{\Delta t} = -\nabla \cdot \mathbf{R}_m^{n+1},
\]