Inertia in Friedmann Cosmologies.

J. Teuber
The Niels Bohr Institute - Blegdamsvej 17, DK-2100 Copenhagen, Denmark

P. G. Hjorth
Department of Physics, University of California - San Diego, La Jolla, CA 92093

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Summary. — Assuming the validity of Mach's principle, we present a formalism allowing the calculation of inertial reaction forces having the mass distribution of an entire Friedmann model as their source. In this scheme, the density parameter characterizing the Friedmann model appears in Newton's second law which in this form can be regarded as a statement about cosmology. We discuss a possible observational consequence and its relation to variable-semester theories.

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1. - Introduction.

The famous principle named after the Austrian philosopher and physicist Ernst Mach (1838-1916) possesses formulations at several levels. It is sometimes stated to express the possibility of drawing inferences, concerning the Universe as a whole, from investigations of local physical phenomena; more precisely, it relates the concept of inertia of a material body to an interaction between the body in question and all other matter in the entire Universe. For instance, the fact that the Universe performs a revolution each 24 hours, as viewed from the Earth, and the fact that the Earth exhibits an equatorial bulge are regarded as cause and effect, respectively.

At the quantitative level, Einstein was the first to suggest that gravitation, as described by general relativity, was the mechanism responsible for this
inertial coupling». On the basis of the linearized or «weak» approximation to GR, it was shown by him that a spherical shell of matter, taken as a model representing (part of) the large-scale universe, may act as a source of forces of inertial or «fictitious» type: when the sphere is accelerated as a whole, a test particle in its centre will be dragged along with the motion; when the sphere rotates, a test particle moving through its centre will feel a Coriolis-like force. This calculation lends strong support to the idea that Mach's principle could be incorporated into GR and that inertial forces could be described as weak gravitational effects. Moreover, since the strength of the coupling between the mass shell and the test particle was given by \( m/r \), \( m \) and \( r \) representing shell mass and radius, respectively, the interaction was long range, like «radiation» in electromagnetic theory. It could thus be expected to produce remarkable effects (from an Olbers' paradox type of argument) if the shell indeed were replaced by the universal distribution of mass. This replacement would also be necessary from the point of view that a comparison between «calculated» and «observed» inertial forces ought to yield an identification of type \( m/r \sim 1 \) \((G=c=1 \text{ is used throughout})\), violating the validity of the weak approximation applied to a single shell of mass.

Unfortunately, the latter generalization implied severe problems. For instance, to describe the acceleration and rotation referred to above, a global Minkowsky metric had to serve as background; this choice, however, introduced the presence of (inertial!) forces tending to destroy the «universe» (for a discussion of this problem, see (1)). Moreover, if the global mass distribution was to be envisaged as rigidly rotating, one would have to accommodate arbitrarily large particle velocities in the formalism.

In this paper, we present a new formalism which circumvents these difficulties. The calculation involving the accelerated sphere is generalized to a (low-density) Friedmann universe and it is shown how the «accelerated» Universe indeed produces the inertial reaction force appearing in Newton's second law. Thus the concept of inertial mass is derived from the formalism. Accordingly, all «masses» and «forces» to be mentioned in the following developments should be regarded as strictly gravitational; for instance, the fundamental «transforming away» by free fall of a force felt in the presence of a large material body should be thought of as a cancellation of two purely gravitational forces, the «inertial» part having cosmos as its source.

In sect. 2, we list the few properties of Friedmann cosmology to be employed later. Section 3 contains a presentation of an «observer-oriented» version of linearized gravity and in sect. 4 this formalism is applied to the case of an observer accelerating through a Friedmann universe. This calculation leads to a somewhat suprising formal determination of the deceleration parameter \( q_0 \).

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