A UNIFIED APPROACH FOR DEALING WITH THE EM SCATTERING FROM SYMMETRIC AND ANTI-SYMMETRIC STRUCTURES

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Abstract It is of both the theoretical and practical importance to reduce the storage and CPU time of moment methods by utilizing the geometrical and physical features of the scatterer. An unified approach based on the group theory is presented to deal with the EM scattering from symmetric and anti-symmetric structures.

Key words EM scattering; symmetric and anti-symmetric structures; generalized image method; group theory; integral equations method

I. Introduction

The requirement of large storage is the main obstruction which restricts the application of moment methods (MM). Making full use of the geometrical and physical features of the scatterer to reduce the storage and CPU time for obtaining the scattered field effectively, is of both theoretical and practical importance[1,3]. In literatures[2,3], it is presented the generalized image method (GIM) can be used for dealing with the EM scattering from symmetric and anti-symmetric structures. In this paper, the general group theory is used to deal with the 3-dimensional EM scattering problem both for symmetric and anti-symmetric structures, and the strictly theoretical basis[4] is given.

II. The General Definition of Symmetry and Anti-Symmetry

The general definition of symmetry and anti-symmetry is described by an operator group $G$ of 8 order

$$G = \{I, R_1, R_2, R_3, R_1R_2, R_1R_3, R_2R_3, R_1R_2R_3\}$$ (1)

The action space of $G$ is $R_3$, which is produced by four basic operators defined as

$$I(x, y, z) = (x, y, z)$$
$$R_1(x, y, z) = (-x, y, z)$$
$$R_2(x, y, z) = (x, -y, z)$$
$$R_3(x, y, z) = (x, y, -z)$$ (2)

Since the coordinate may be selected arbitrarily, in fact, there are typical subgroups to be contained in $G$ as follows:
With the help of Eq. (3), a general definition of symmetry and anti-symmetry can be given:

**Definition** Let the dielectric constant and magnetic permeability of a structure be \( \varepsilon(\mathbf{r}) \) and \( \mu(\mathbf{r}) \); with the coordinate selected appropriately, the following equation is valid at least for arbitrary subgroup \( H \) in Eq. (3):

\[
\begin{align*}
\varepsilon[T(x, y, z)] &= \varepsilon(x, y, z) \\
\mu[T(x, y, z)] &= \mu(x, y, z), \quad (\forall T \in H)
\end{align*}
\]

Let the subgroup of the highest-order which satisfies Eq. (4) be \( H_M \). Then the structure is called symmetric when \( H_M = H_iS_i(1 \leq i \leq 3) \), and the structure is called anti-symmetric when \( H_M = H_iA_i(1 \leq i \leq 4) \). If \( H_M = HSA \), the structure is called the complex symmetric. \( H_M \) is called the characteristic subgroup.

It is noted that, the main purpose of adopting the term "structure" is to emphasize that the scatterer can be either single, or a component object.

### III. Approach to Integral Equation

In the following paragraph, the problem of preliminary approach to volume integral equation by the aid of GIM is studied in order to reduce the storage and CPU time which is required in solving the equation.

Let the magnetic permeability of scatterer be equal to that in the free space \( \mu_0 \), dielectric constant is \( \varepsilon(\mathbf{r}) \), so, the integral equation of electric scattering field can be written as

\[
\begin{align*}
\mathbf{E}'(\mathbf{r})/(j\omega) + \int_V (\varepsilon_0 - \varepsilon) \mathbf{E}'(\mathbf{r}') \cdot \mathbf{G}(\mathbf{r}'|\mathbf{r}) d\mathbf{v} &= \int_V (\varepsilon - \varepsilon_0) \mathbf{E}'(\mathbf{r}') \mathbf{G}(\mathbf{r}'|\mathbf{r}) d\mathbf{v}, \quad (\mathbf{r} \in V)
\end{align*}
\]

where \( \mathbf{E}_i \) is the incident electric field, \( \varepsilon_0 \) is the dielectric constant of free space, \( \mathbf{G}(\mathbf{r}'|\mathbf{r}) \) is the dyadic Green's function of electric type, \( \omega \) is the angular frequency, and \( V \) is the entire space occupied by the scatterer.

Let \( H \) be the characteristic subgroup of scatterer, \( V_1 \) the basic element of \( V \), and then