Remarks on the Relativistic Quantum Gas with Interaction and Comparison with the Bootstrap Hypothesis.

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Summary. — In this paper we discuss some consequences of the Bernstein, Dashen and Ma formalism of statistical mechanics, concerning the definition of the mass spectrum and the level density for interacting gas. A recipe for constructing the relativistic level density of N interacting quantum particles is provided. Comparison with the bootstrap equation leads to a dynamical interpretation of the Yellin coefficients.

1. — Introduction.

Relying on the Bernstein, Dashen and Ma (ref. (1)) formalism of quantum relativistic statistical mechanics we discuss the general definition of both the level density $\omega(P^2)$ and the mass spectrum $\sigma(P^2)$ for a portion $P^2$ of interacting hadronic matter. These two functions receive contributions from states with any number of particles which are both bosons and fermions. Yet, to give a meaning to the concept of statistical decay of hadronic systems we need the definition of the level density at a fixed number of particles. By taking all those graphs contributing to $\omega(P^2)$ which have a fixed number of external legs, the latter is provided.

On the other hand, recently Chaichian, Hagedorn and Hayashi (ref. (2)) succeeded in expanding the free level density of bosons and fermions at a fixed

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(2) M. Chaichian, R. Hagedorn and M. Hayashi: CERN TH. 1975.

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number of particles into cluster contributions (*). We show how this cluster expansion can be interpreted in terms of the free-particle mass spectrum, obtaining a formula which can be generalized to the case where both the mass spectrum and the level density have interaction contributions.

These formal results enable us to discuss the general definition of statistical decay and to inquire under what hypotheses a bootstrap equation for the mass spectrum results. As an outcome we get a dynamical interpretation of the Yellin coefficients and the definition of a wider class of models in which the mass spectrum can be expanded in series of ordinary phase-space integrals. The bootstrap models are particular cases in this class and we feel that an investigation of the general case could be useful. We leave this anyway for future work.

2. - General definitions for the quantum relativistic gas.

We need to write a list of basic definitions; we assume that the thermodynamical system we want to describe statistically is enclosed in a box of volume $V$. The mechanism responsible for the confinement of the system is not specified.

The frame of reference in which the box of volume $V$ is at rest has a four-velocity $u^\mu$

\begin{equation}
  u^\mu u_\mu = 1.
\end{equation}

We introduce a four-vector volume $Q^\mu$ defined as follows:

\begin{equation}
  Q^\mu = \frac{2V}{(2\pi)^3} u^\mu.
\end{equation}

Now we suppose that the system which we want to describe statistically is governed by a microscopic quantum relativistic dynamics which gives us the rules for constructing the Hilbert space of its states and the observables defined in it. In particular we are interested in the four-momentum operator

\begin{equation}
  \hat{P}^\mu.
\end{equation}

Moreover we suppose that the free states of the lowest-mass particles form a complete set. This is the asymptotic condition necessary to build up the scattering theory. Therefore a basis of our Hilbert space is given by

\begin{equation}
  |p_1, p_2, \ldots, p_N\rangle = |p_1\rangle \cdots |p_N\rangle,
\end{equation}

(*) We are grateful to R. Hagedorn for letting us know about these results before publication.