Summary. — We analyse quantitatively, in the frame of the usual relativistic theories of continuum media and electrodynamics, a simplified version of the Trouton-Noble experiment. It is checked at first, in the \( \nu^2/c^2 \) approximation as well as in the exact form, that the negative result of the experiment is a consequence of the application of the angular-momentum theorem to the electromagnetic subsystem. An analogous result is obtained when analysing the mechanical subsystem. The apparent paradox can be also explained by considering the angular-momentum conservation law of the closed total system. Finally, energy considerations of the closed total system lead, obviously, to the same conclusion.

1. — Introduction.

It is well known that one of the experiments to verify the Earth motion through the ether was that proposed by Trouton (1) in 1902, and its negative result has been considered as a proof in favour of the special theory of relativity.

The original idea is the following: when a charged plane-plate capacitor is moving with speed \( v \) with respect to the ether, if \( v \) is parallel to the plates, there exists a magnetic field \( B \), besides the electric field \( E \) between them, such that the electromagnetic energy

\[
U_1 = \frac{1}{2} \int \left( \varepsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \, dv = U_2 (1 + \beta^2),
\]

where $U_e$ is the energy of the electric field alone, $\beta = v/c$ and $c$ the speed of light. If $v$ is orthogonal to the plates, the electromagnetic energy is $U = U_e$, leading to Trouton's conclusion that, when a capacitor is hang up, moving freely near the Earth's surface (and consequently being dragged with it in its motion through the ether), it will rotate until the energy attains a minimum, i.e. the capacitor plates become orthogonal to $v$.

An alternative analysis was proposed by Searle (2) by considering the torques and can be found in the Panofsky and Phillips (3) book. It is as follows. If we consider a pointlike charge $+q$ in one of the plates and its image $-q$ in the other, each one moves in the electromagnetic field of the other, being acted, respectively, by equal but opposite forces with different lines of action, producing consequently a torque that will rotate the capacitor until its plates become orthogonal to the speed.

If one considers the usual relativistic electrodynamics, a torque still appears, giving rise to an apparent paradoxical situation, since the experiment shows that the capacitor does not rotate.

The problem has been analysed from different points of view. Butler (4) has proposed a new definition of the electromagnetic-energy density, which implies that the electromagnetic energy of the capacitor is independent of its orientation with respect to the velocity; Strnad (5) gives an interpretation in terms of the virtual-work principle; Gron (6) considers the problem as an example to which the asynchronous formulation of relativistic statics can be applied, and Furry (7) thinks that it is a problem in which a hidden momentum, required by the relativity theory, explains the null result of the experiment in the way of reasoning of Pauli (8).

The aim of the present work is to show by explicit calculation how the usual relativistic theories of electromagnetism and continuum media allow one to completely explain the apparent paradox. In sect. 2, a simplified version of the problem will be stated, by using instead of a capacitor a system consisting of two opposite electric charges joined by a nonconducting rod. In sect. 3 a solution in the $\beta^2$ approximation will be given. In sect. 4 the exact expression of the angular momentum associated to the electromagnetic field will be obtained by direct calculation, checking that its time derivative matches with the external torque that acts on the electromagnetic subsystem. An analogous answer to the paradox, but concerning this time the mechanical subsystem, will

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