k-Dependent Lagrangian and Dispersion Relation in Vacuum Electrodynamics (*)(**).

A. Holz (***)
Departamento de Física, Universidad de Chile - Santiago

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Summary. — The Lagrangian $L$ of an electromagnetic field $(e, b)$ in vacuum in the presence of a strong static electromagnetic field $(E, B)$ is studied from the symmetry point of view. From $(e, b)$, $(E, B)$ and $(k, i\omega)$ quantities are formed which are invariant under the transformations of the improper Lorentz group and charge conjugation. Without static field the additional term in $L$ of the form $\omega^2 e^2 + k^2 b^2 - (k \cdot e)^2 + 2i\omega e \cdot (k \times b)$ leads to solutions of the Maxwell equations where $m^2 k^2 = 0$ is not satisfied for all branches.

1. - Introduction.

The existence of sources of strong electromagnetic fields led in recent years to an enhanced interest in nonlinear vacuum electrodynamics (see for instance (1,2)). While the majority of authors considers the low-energy range, in the following possible effects are considered which could occur for highly energetic radiation. The theory developed is based on symmetry considerations only. This implies that no order of magnitude of the quantities entering the Lagrangian $L$ can be given nor can be decided if they are necessarily different from zero.

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(***) Present address: Instituto de Física, Universidade Federal do Rio Grande do Sul, Porto Alegre.

In Sect. 2 we consider the Lagrangian of an electromagnetic plane wave \((e, b)\) with four-wave-vector \(k\) in the presence of uniform electro- and magnetostatic fields \((E, B)\). It is understood that the problem has translational invariance which allows solutions in terms of plane waves. \(L\) is required to be form invariant under the transformations of the improper Lorentz group, and charge conjugation. The \(k\)-dependence of \(L\) is taken into account but no general expression of \(L\) in terms of all possible invariants which can be constructed from \((e, b), k, (E, B)\) is given. The \((e, b)\) field is considered only up to quadratic terms in \(L\). Higher-order derivatives or \(k\)-dependence of the Fourier-transformed Lagrangian enters the theory because the other field variables coupled to the electromagnetic field are assumed to be eliminated.

In Sect. 3 the constitutive equations for the propagation of a plane wave without static fields are given. A dispersion relation is derived. It is suggestive that this effect might be of importance for highly energetic photons where it is assumed that the relation \((c = 1)\)

\[
k \cdot k - \omega^2 = 0
\]

is not necessarily satisfied because it is a consequence of

\[
d = e, \quad b = h.
\]

The last equations, however, are special constitutive equations which apply to the free electromagnetic field only. The idea we have in mind is that Maxwell’s equations are of general validity, whereas the constitutive equations depend on the interaction of the electromagnetic field with other fields or sources.

2. - Construction of the invariants.

We use the field tensors of the time varying and static fields in the form

\[
F = \begin{pmatrix}
0 & b_3 & -b_2 & -ie_1 \\
-b_3 & 0 & b_1 & -ie_2 \\
b_2 & -b_1 & 0 & -ie_3 \\
ie_1 & ie_2 & ie_3 & 0
\end{pmatrix},
\]

\[
\bar{F} = \begin{pmatrix}
0 & B_3 & -B_2 & -iE_1 \\
-B_3 & 0 & B_1 & -iE_2 \\
B_2 & -B_1 & 0 & -iE_3 \\
iE_1 & iE_2 & iE_3 & 0
\end{pmatrix},
\]