A Four-Dimensional Green's Function Approach to the Calculation of Gravitational Radiation from a Particle Falling into a Black Hole (*)

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(ricevuto il 24 Ottobre 1972)

Summary. — A four-dimensional Green's function method to calculate the gravitational radiation from a particle moving in the field of a black hole is described in detail. This technique is applicable even to problems in which the background geometry has no symmetry. An explicit example of the Schwarzschild black hole is worked out, and a comparison of the projection of our results onto definite angular momenta shows excellent agreement with the literature.

1. — Introduction.

In recent years, theoretical investigations have been made to calculate gravitational radiations from a particle falling into a Schwarzschild black hole (1,2). The approach of RUFFINI, ZERILLI, WHEELER and co-workers utilizes the spherical symmetry of the background geometry to decompose the metric tensor into tensor harmonics; it is therefore not suitable for more general cases, including the Kerr background.

In this work we would like to examine an alternative approach, based on the Green's function technique, which does not require decomposition into definite angular-momentum states and may be useful in handling cases where

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

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symmetry is lost. The four-dimensional Green's function method for solving Einstein's equations has been considered in a different context by Sciama, Waylen and Gilman (3), and by DeWitt and Brehme (4). However, this method has been beset by many difficulties and practical applications have been minimal so far (5). We will treat the general formalism for gravitational radiation caused by the motion of a particle in a background field, considering the effect of the particle motion as a perturbation on the background geometry. By means of the Green's function technique utilizing bitensors, it is possible to reduce the original second-order partial differential equations to a system of first-order ordinary differential equations. The boundary conditions associated with the black holes are readily taken care of. Specifically, as a check on the validity of our formalism, we shall compute the radiation field for a particle falling into the Schwarzschild black hole radially from spatial infinity.

The agreement of our numerical results with previous calculations are satisfactory, and the extension of our work to the Kerr black-hole regime will be discussed elsewhere.

2. - General formalism.

When the infalling mass ($m$) is much smaller than the mass for the black hole ($M$), the perturbation on the background space satisfies the following equations as can be deduced from Einstein's equations:

\[ h_{\alpha \beta; \gamma} = - \left( f_{\alpha ; \beta} - f_{\beta ; \alpha} \right) + 2R_{\alpha \beta}^{\gamma \delta} h_{\gamma \delta} + h_{\gamma ; \gamma, \alpha \beta} + g_{\alpha \beta} \left[ f_{\gamma ; \gamma} - h_{\gamma ; \gamma, \gamma} \right] = -2\pi \delta T_{\alpha \beta}, \]

where $h_{\alpha \beta}$ is the perturbation on the background metric, i.e.

\[ g_{\alpha \beta \text{ perturbed}} = g_{\alpha \beta \text{ background}} + h_{\alpha \beta}, \]

and $\delta T_{\alpha \beta}$ represents the perturbation in the energy-momentum tensor due to the infalling particle, while $f_{\gamma} = h_{\gamma ; \gamma}$. In the above equation we have assumed that the background space has a constant curvature, i.e. $R_{\alpha \beta} = 0$, and we have neglected the radiation reaction; this is justified as long as $m \ll M$. The covariant derivatives are taken against unperturbed background space-time.

In the particular gauge

\[
\begin{cases}
    h_{\alpha \beta, \gamma} = 0, \\
    h_{\alpha} = 0,
\end{cases}
\]