Inflationary solutions in general scalar tensor theory of gravitation

A BANERJEE, S B DUTTA CHOUDHURY, N BANERJEE and A SIL
Relativity and Cosmology Research Centre, Department of Physics, Jadavpur University, Calcutta 700032, India

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Abstract. Extended inflation solution in Brans–Dicke theory given by Mathiazhagan and Johri (MJ) is shown as the unique solution only if the scale factor is assumed to be a power function of the scalar field. Only the consistent solution amongst the set of solutions given by Patra, Roy and Ray is found identical to the MJ solution. Both exponential inflation and power function inflation are studied in general scalar tensor theory where the parameter \( \omega \) is a function of the scalar field. It is noted that exponential inflation is forbidden in Brans–Dicke theory where \( \omega \) is a constant.

Keywords. Cosmology; inflation; scalar tensor theory.

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1. Introduction

Extended inflation given by La and others [1, 2] was based on the inflationary solutions obtained by Mathiazhagan and Johri [3] in Brans–Dicke theory of gravity [4]. Brans–Dicke theory with constant \( \omega \) parameter is ruled out in view of the recent light deflection and time delay experiments [5] supporting values of \( \omega > 500 \). Such theories in fact lead to Einstein’s theory of gravity in the limiting case of \( \omega \to \alpha \). It is therefore worthwhile to consider a more general scalar tensor theory [6, 7, 8] where \( \omega \) is a said to be a function of the scalar field \( \varphi \) and thus varies with time. It is worthwhile to mention here that limitations of extended inflation had been discussed by Weinberg [9].

In §2 it has been shown that the well known solution of Mathiazhagan and Johri (hereinafter referred to as MJ) [3] is the unique solution for vacuum energy only when the scale factor \( R(t) \) is assumed to be a power function of the scalar field \( \varphi \). It is further shown that the other solutions in Brans–Dicke theory given by Patra et al [10] are not consistent with the field equations. One of them satisfies the field equations only in a special case but it is exactly the same as MJ solution and hence cannot be claimed to be a new solution.

In §3 we have shown that the old inflationary solution given in the form of an exponential function of time is fully consistent with a variable \( \omega \) but is not admissible in any case in the BD theory. We have given an exact solution of this type in the general scalar tensor theory where the function \( \omega \) increases with time and may be quite large in the present era.

At the end we present another particular solution with the scalar factor \( R \) growing not exponentially but rather as power function of time. The parameter \( \omega \) found in
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this case, however, does not enable us to draw any definite conclusion about its time behaviour throughout.

2. Field equations and discussion of solutions with constant $\omega$

The general scalar tensor theory with $\omega = \omega(\phi)$ was originally proposed by Bergmann [6], Wagoner [7] and Nordtvedt [8]. The action in that case is

$$S = \int d^4x \sqrt{-g} \left( -\phi R - \frac{\omega}{\phi} g^{\mu\nu} \phi_\mu \phi_\nu + 16\pi L_m \right)$$  \hspace{1cm} (1)

where $\phi$ is the scalar field, $\omega$ is a parameter, being in general an arbitrary function of time and $L_m$ includes a Higgs type sector which undergoes a strongly first order phase transition at high temperatures. The gravitational field equations corresponding to action (1), with the spatially flat ($k = 0$) Robertson-Walker metric, are

$$\frac{R^2}{8\pi \rho_v} - \frac{\omega}{6} \phi^2 - \frac{R'}{R} \phi' = \frac{8\pi \rho_v}{\phi} - \omega \left( \frac{\phi'}{\phi} \right)^2 - 2 \frac{R'}{R} \phi' - \frac{\phi''}{\phi}$$ \hspace{1cm} (2)

and the scalar wave equation for the scalar field is

$$\frac{\phi''}{\phi} + 3 \left( \frac{R'}{R} \phi' \right) = \frac{1}{2(\omega + 3)} \left[ 32\pi \rho_v - \phi^2 \frac{d\omega}{d\phi} \right]$$ \hspace{1cm} (3)

Returning to the field equation (2) one has

$$\frac{2}{R} + 4 \frac{R'}{R^2} = \frac{16\pi \rho_v}{\phi} - \frac{5}{R} \frac{R'}{\phi'} - \frac{\phi''}{\phi}.$$ \hspace{1cm} (5)

If we now assume $R^3 = \phi^2$, that is, the scale factor as a power function of $\phi$, (5) on integration yields the first integral

$$\phi'' = \frac{2C}{(2\alpha + 1)} \phi + D\phi^{-2\alpha}$$ \hspace{1cm} (6)

with the constant $C = 16\pi \rho_v/(2\alpha(3 + 1)(\alpha \neq 3/2)$ and $D$ being an integration constant. The solution for the scalar field $\phi$ may be obtained after second integration, we have

$$t = \int \frac{d\phi}{\sqrt{[(2C/(2\alpha + 1))\phi + D\phi^{-2\alpha}]^{1/2}}}.$$ \hspace{1cm} (7)

Returning to the field equation (2) one has

$$\left( \frac{\phi^2}{9} + \frac{\alpha}{3} - \frac{\omega}{6} \right) \frac{\phi^2}{\phi} = \frac{8\pi \rho_v}{3} = \text{constant.}$$ \hspace{1cm} (8)