Fields Without Partners.

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Summary. — It is proved that field operators characteristic of ultra-local quantum-field theories do not possess conjugate partner fields fulfilling canonical commutation relations in the Weyl form. Our analysis, which is based on a study of the expectation functionals, eliminates any possible partners whatsoever, not only the ones suggested by dynamical considerations.

1. Introduction.

A common and reasonable assumption in quantum field theory regards the existence of a field \( \phi(x) \) and its partner (conjugate) \( \pi(x) \) which fulfill the familiar commutation relation

\[
[\phi(x), \pi(y)] = i \delta(x - y).
\]

Often this is "dressed up" by the Weyl form of the commutation relation

\[
V(g) U(f) = U(f) V(g) \exp [i(f, g)], \quad (f, g) = \int f(x)g(x)dx,
\]

where \( f(x) \) and \( g(x) \) are suitable real test functions. Minimal (ray) continuity arguments imply

\[
U(f) = \exp [i\phi(f)], \\
V(g) = \exp [i\pi(g)]
\]
for the smeared self-adjoint fields, e.g.

\[ \varphi(f) = \int \varphi(x) f(x) \, dx. \]

Although this formulation is more satisfactory mathematically, the physical content is essentially the same as in the heuristic commutation relation. Representations of the field and its partner can be characterized by the expectation functional

\[ E(f, g) = \langle \Phi, U(f) V(g) \Phi \rangle \]

for some cyclic vector \( \Phi \in \mathcal{H} \), the Hilbert space of the representation. In cases where \( \Phi \) is a cyclic vector for the field \( \varphi(x) \) alone much can be learned from just a study of

\[ (1.1) \quad E(f) = \langle \Phi, U(f) \Phi \rangle. \]

In particular, Araki (1) has shown that the matrix elements

\[ (1.2) \quad (U(f') \Phi, \pi(g) U(f) \Phi) = \frac{1}{2} \langle g, f' + f \rangle E(f - f') \]

are direct consequences of the relation \( E(f, -g) = E(f, g) \exp[i(f, g)] \) (e.g., a time-reversal invariance of \( \Phi \)) and the Weyl commutation relation. It would seem from (1.1) and (1.2) that the existence of \( \pi(x) \) is almost assured by the existence of \( \varphi(x) \). However, this is by no means the case as we shall explicitly show for a certain fairly large class of field representations.

It has been shown elsewhere (2) that ultralocal scalar quantum fields are characterized by expectation functionals of the form

\[ (1.3) \quad E(f) = \exp \left[ -\int dx \left\{ 1 - \cos(\lambda f(x)) \right\} \sigma^2(\lambda) \, d\lambda \right]. \]

Here \( \sigma(\lambda) \), the model function, is a real, even function of \( \lambda \in \mathbb{R} \) that must fulfill the condition

\[ (1.4) \quad \int \frac{\lambda^2}{1 + \lambda^2} \sigma^2(\lambda) \, d\lambda < \infty, \]

but need not be \( L^1(\mathbb{R}) \). Clearly, if (1.4) is fulfilled, then \( E(f) \) is well defined (i.e. not zero) for all real \( f \in C_0^\infty \), or all piecewise constant functions with compact support, etc., and \( E(f) \) is completely characterized by its values for such classes of elements. For fixed \( f(x) \) the function \( C_f(s) = E(sf) \) is a positive-

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