Mechanism and control of convective heat transfer

---Coordination of velocity and heat flow fields

GUO Zengyuan

Department of Engineering Mechanics, Tsinghua University, Beijing 100084, China

Abstract A second look has been given at the mechanism of convective heat transfer based on the analogy between convection and conduction with heat sources. The strength of convective heat transfer depends not only on the fluid velocity and fluid properties, but also on the coordination of fluid velocity and heat flow fields. Hence, based on the included angle of velocity and temperature gradient vectors, the presence of fluid motion may enhance or reduce heat transfer. With this concept, the known heat transfer phenomena may be understood in a deeper way. More important is that some novel approaches of heat transfer control can be developed.

Keywords: convective heat transfer, coordination, heat transfer enhancement.

Convective heat transfer is one of basic ways of heat transport, which is essentially conducted under fluid motion. Because fluid motion itself carries heat, convection is usually considered to be much more efficient in heat transport than conduction\(^1,2\). In view of the fact that convective heat transfer is of broad application in various engineering areas, a large amount of work has been done on detailed study of convective heat transfer in the past decade. However, the conventional way to investigate convective heat transfer is first to classify convection into internal/external flow, forced/natural convection, boundary layer flow or rotating flow, etc. Then, attention in both analytical and experimental study is always concentrated on the discussion of convective heat transfer coefficient, \(h\) and the corresponding dimensionless parameter, \(Nu\). Finally, \(Nu\) is expressed as different kinds of functions of Reynolds number, \(Re\) (or Grashof number, \(Gr\)) and Prandtl number, \(Pr\)\(^3,4\). It can be easily found from those known correlations that \(Nu\) is dependent on the fluid velocity and fluid properties (capacity, density, conductivity, viscosity, etc) as well as the form and state of fluid flow.

In this note, starting from energy equation of convection, a second look at the mechanism of convective heat transfer is given from the angle of coordination of velocity and heat flow fields. With this concept, a number of heat transfer phenomena can be understood in a better way. More important is that some new approaches of heat transfer control will be developed based on the concept of field coordination.

1 Analog of convection and conduction

Consider the energy equation for 2-D boundary-layer steady flow over a cold flat plate at zero incident angle,

\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right), \tag{1}
\]

where \(\rho\), \(c_p\) and \(k\) are density, specific heat and conductivity of fluid, respectively; \(T\) is fluid temperature; \(u\) and \(v\) are velocity components in the \(x\)- and \(y\)-direction.

Energy equation for conduction with heat source \(\dot{q}\) is

\[
\dot{q} = \frac{\partial}{\partial y} \left( k \frac{\partial T}{\partial y} \right). \tag{2}
\]

It is easy to find that the convection term in the energy equation for the boundary layer flow corresponds to the heat source term in the heat conduction equation. The difference is that “heat source” term in convection is the function of the flow field. The presence of heat sources leads to the increase of heat flux at the boundary for both the conduction and convection problems. The integral of eq. (1) leads to

\[
\int_0^\delta \rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) dy = -k \left. \frac{\partial T}{\partial y} \right|_w, \tag{3}
\]

where \(\delta\) is the thermal boundary layer thickness, and \(R\) represents the duct radius.

Eq. (3) indicates that the wall heat flux is equal to the overall strength of heat sources inside the thermal boundary layer. This implies that the convective heat transfer can be enhanced by raising the value of the integral of convection terms (heat sources) over the thermal boundary layer/duct radius. On the opposite, the reduction in source strength will result in a decrease of heat transfer rate.

The energy equation in general form for heat convection is given as follows:

\[
\rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) = \frac{\partial}{\partial y} \left( k \frac{\partial^2 T}{\partial y^2} \right) + \dot{q}. \tag{4}
\]

Again, the convection can be regarded as conduction with heat sources. Then, we have

\[
\int_0^\delta \rho c_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right) dy - k \left. \left( \frac{\partial^2 T}{\partial y^2} \right) \right|_w - \dot{q} \left. \frac{\partial T}{\partial y} \right|_w, \tag{5}
\]

There are three kinds of heat sources: real heat source \(\dot{q}\), convection induced equivalent heat source, and multi-dimensional conduction induced equivalent.
heat source. For example, the heat transfer coefficient becomes much larger for the hot flow over cold plate boundary layer with exothermic chemical reaction or for the humid air flowing over the hot surface. As the same, the multi-dimension term in eq. (4) stands for the impact of 2-D or 3-D conduction effect on the heat transfer coefficient.

2 Coordination of velocity and heat flow fields

Eq. (3) can be rewritten with the convection term in the vector form:

$$\int \rho c_p (U \cdot \nabla T) dy = -k \frac{\partial T}{\partial y},$$

(6)

Introduce the following dimensionless variables,

$$\bar{U} = \frac{U}{U_x}, \quad \nabla \bar{T} = \left(\frac{\nabla T}{(T_x - T_w) / \delta_t}, \bar{y} = \frac{y}{\delta_t}, T_x > T_w.\right)$$

Then, the dimensionless form of eq. (6) is given as

$$\bar{N}u_x = \bar{R}e_x \bar{P}r \int (\bar{U} \cdot \nabla \bar{T}) d\bar{y},$$

(8)

where \(\bar{N}u_x\), \(\bar{R}e_x\), and \(\bar{P}r\) represent Nusselt number, Reynolds number and Prandtl number respectively. The vector dot product, \(\bar{U} \cdot \nabla \bar{T}\) in eq. (8) can be expressed as

$$\bar{U} \cdot \nabla \bar{T} = \bar{U} \cdot \nabla \bar{T} \cos \beta,$$

(9)

where \(\beta\) is the included angle between the velocity vector and the heat flow vector.

It is well known that Nusselt number depends on Reynolds number and Prandtl number intensively. However, it can be seen that Nusselt number also depends on the included angle of velocity and temperature gradient vectors. The heat transfer coefficient will reach its maximum if \(\beta = 90^\circ\). The smaller the included angle is, the greater the heat transport rate becomes for \(\beta < 90^\circ\).

The further derivation of formula (8) obtains

$$St = \frac{\bar{N}u_x}{\bar{R}e_x \bar{P}r} = I \int (\bar{U} \cdot \nabla \bar{T}) d\bar{y},$$

(10)

where \(St\) is the function of \(Re\), \(Pr\), and usually less than unity. It can be seen from formula (9) that we have two vector fields, \(\bar{U}\) and \(\nabla \bar{T}\) or three scalar fields \(\bar{U}, \bar{V}, \bar{W}\), and \(\cos(\beta)\). Hence, the strength of convective heat transfer depends not only on the velocity, temperature difference and fluid properties, but also on the coordination of fluid flow and heat flow fields. The heat source term in formula (8) cannot be of large value unless local values of those scalar fields are simultaneously large. Assuming that \(\bar{U}\) and \(\nabla \bar{T}\) are uniform, and their included angle, \(\beta\), is equal to zero everywhere in the whole field, then

$$St = 1,$$

namely,”

$$Nu_x = \bar{R}e_x \bar{P}r.$$