Co-ordinates for Fermions (*).

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Summary. — We present an examination of the Dirac matrices as spin co-ordinate. The field depending on the Minkowski space-time $x$ and the Dirac matrices $\gamma$ is introduced. The orbital and spin parts of the angular momentum appear on an equal footing when $x$ and $\gamma$ are Lorentz transformed simultaneously.

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1. – Introduction.

The idea of using «anticommuting c-numbers» to deal with fermions has appeared a few times in the history of high-energy physics. The first work introducing anticommuting fields to describe fermions in the context of the functional method was given by MATTHEWS and SALAM (1). RZEWUSKI (2) and BEREZIN (3) took similar approaches in their field theory. Such an unfamiliar idea, however, naturally faced a criticism. Showing quantization of spinor fields in terms of ordinary c-numbers, KLAUDER (4) contended that the

(*) To speed up publication, the author of this paper has agreed to not receive the proofs for correction.

(2) J. RZEWUSKI: Field Theory II (Warsaw, 1964).
use of anticommuting e-numbers was «an unnecessary addition to mathematical physics».

More recently, the concept of anticommuting e-numbers appeared in the fermionic dual theory (9). GERVAIS and SAKITA (8) discussed possible new invariances in the generalized dual model and IWASAKI and KIKUKAWA (7) presented the fermionic string employing anticommuting fields. The interest in anticommuting e-numbers was duly enhanced with the presentation and development of the theory of supersymmetry in the past several years (9). Among other aspects of the theory, what is particularly new is the introduction of the superspace. In this framework, the field operators depend on anticommuting Majorana spinors as well as on the usual Minkowski space-time.

The aim of this paper is to examine the use of Dirac matrices as spin co-ordinate (9). We go back to the Dirac equation and let the field depend on the $\gamma$-matrices as well as on Minkowski space-time without destroying the original meaning of the equation. This is shown in sect. 2. In sect. 3, the Lorentz transformation properties of this matrix function are examined. Here we will see that the orbital and spin parts of angular momentum appear on an equal footing under the Lorentz transformation of both $x$ and $\gamma$. Finally, in sect. 4 we discuss the co-ordinate-like nature of the Dirac matrices.

2. – Alternative approach to the Dirac equation.

In the usual approach to the Dirac equation

\begin{equation}
(2.1) \quad \left( i \gamma^\mu \frac{\partial}{\partial x^\mu} - m \right) \psi(x) = 0
\end{equation}

with

\begin{equation}
(2.2) \quad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu},
\end{equation}

one regards $\psi(x)$ as a 4-component column matrix. However, this does not seem to be the only way to consider the equation. We may let $\gamma$ be a $4 \times 4$ matrix without altering the content of the equation. Since any $4 \times 4$ matrix can be written as a linear combination of the Dirac matrices and their products,