Scattering Problem of the Lorentz-Dirac Equation: Phenomena of Quasi-Confinement of Dirac’s Monopoles.

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Summary. — The scattering problem of the Lorentz-Dirac equation is investigated. In particular, the Coulomb scattering of monopoles and also the monopole–charged-particle scattering are considered in detail. In contrast to the cases of the ordinary scatterings without the radiative friction term, it is found that in the scattering process there exists an upper bound of momentum transfer, which is independent of the incident energy and the impact parameter. It is pointed out that such a property of the momentum transfer implies a kind of confinement. In particular, a composite system of Dirac’s magnetic monopoles of the opposite sign cannot be ionized by any single collision process, as long as the binding energy exceeds a critical value, which is determined by the aforementioned upper bound of the momentum transfer.

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1. - Introduction.

In contrast to the equation of motion in the external force field $f_{\text{ext}}^\mu$ of a neutral particle, which is $m a^\mu = f_{\text{ext}}^\mu$, a charged particle satisfies the Lorentz-Dirac equation (1)

$$a^\mu = \frac{1}{m} f_{\text{ext}}^\mu + \tau_0 \left( \dot{a}^\mu - \frac{1}{c^2} (a^\sigma a_\sigma) u^\mu \right),$$

(1.1)

where

\begin{equation}
\tau_0 = \frac{2}{3} \frac{e^2}{m_0^2},
\end{equation}

and \( u^\mu(\tau) \), \( a^\mu(\tau) \) and \( \dot{a}^\mu(\tau) \) are the velocity, acceleration and the derivative of \( a^\mu(\tau) \) with respect to the proper time \( \tau \), respectively. The last term of eq. (1.1) is the radiative friction term, which relates to the loss of the energy-momentum of the charged particle in the form of the radiation. Equation (1.1) was derived by eliminating the electromagnetic field \( F_{\mu\nu}(x) \) from the well-known equations of electrodynamics

\begin{equation}
\partial_\tau F_{\mu\nu}(x) = e \int u^\mu(\tau) \delta^4(x - z(\tau)) \, d\tau,
\end{equation}

\begin{equation}
m\ddot{z}^\mu(\tau) = e F_{\mu\nu}(z) u^\nu(\tau) + j^\mu_{\text{ext}},
\end{equation}

and the energy-momentum conservation of the whole system. Equations (1.1) and (1.2) are valid also for the magnetic monopole \( ^2 \) if we make a trivial change of \( e^2 \) by \( ^* e^2 \) in the definition of \( \tau_0 \).

When we consider the scattering of charged particles, it is customary to use eqs. (1.3) and (1.4), or their quantum version. That is, we start from the scattering of the neutral particles as the zeroth approximation and then include the effects of the "bremsstrahlung" as the perturbation step by step. However, such a perturbational calculation is not adequate when we consider, for example, the scattering of magnetic monopoles. Since in the Lorentz-Dirac equation all the terms of the "bremsstrahlung" are considered simultaneously, it is desirable to adopt the solutions of the Lorentz-Dirac equation as the zeroth approximation of the scattering problems. In the field theory, we know that when infinitely many graphs of the perturbation expansion are summed up, a qualitatively new feature often arises, which is not shared by each term of the expansion. For example, if we solve the Bethe-Salpeter equation \( ^\text{(*)} \), the bound-state poles come to appear in the complex energy plane of the scattering amplitude, which do not exist in each term of the expansion. Another example is the appearance of the asymptotic freedom \( ^\text{(4)} \) for some types of Lagrangians. Therefore, it is not surprising if a qualitatively new feature

