A Unified Description of the Charge Structure of the Nucleon.

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Summary. -- A unified description of the charge structure of the nucleon is deduced from a nonlinear Bopp-Pais-Uhlenbeck equation.

Twenty-five years ago CLEMENTEL and I proposed an empirical model of the charge structure of the proton; the invention of the model was stimulated by the analysis of the pioneering experiments undertaken by HOFSTADTER on the elastic scattering of high-energy electrons by protons (1,2). Let \( q_p(r) \) be the charge density of the proton and \( F_p(q^2) \) the corresponding form factor. The analytical description of the model is

\[
\begin{align*}
(1.1a) & \quad q_p(r) = (\eta m_p^2/4\pi) Y(m_p r) - (\eta - 1) \delta(r), \\
(1.1b) & \quad \int q_p(r) \, dr = 1, \\
(1.2a) & \quad F_p(q^2) = \{m_p^2 + (1 - \eta) q^2\} / (q^2 + m_p^2), \\
(1.2b) & \quad F_p(0) = 1, \\
(1.2c) & \quad \{dF_p(q^2)/dq^2\}_{q^2=0} = - (\eta/m_p^2),
\end{align*}
\]

where $\eta$ is a number, $m_p$ a parameter homogeneous to the inverse of a length and $Y(m_p r)$ the Yukawa function

$$Y(m_p r) = \exp \left[ -\frac{m_p r}{r} \right] = y(m_p r)/r .$$

The analyses of the data carried out by Hofstadter and collaborators led to the following results:

$$\eta = 1.2 , \quad m_p = (6\eta/r^2_p)^{\frac{1}{3}} = 4.69\mu ,$$

where the root-mean-square radius of the proton is assumed to be

$$r_p = \sqrt{\langle r^2 \rangle_p} = 0.8 \cdot 10^{-13} \text{ cm}$$

and $\mu$ is the inverse Compton wave-length of the pion. The unusual feature of the model lies mainly in the parameter

$$f = \eta - 1 ,$$

which determines the amount of negative charge placed in the Dirac singularity at the centre of the proton and also defines the compensating positive charge distributed in a Yukawa-type cloud (2): if $\eta = 1$, one has the Hofstadter model, whereas, if $\eta = 0$, the proton is described as a pointlike source of the electromagnetic field. It has been found that model (1) corresponds closely to the experimental form factor of the nucleon in the interval $0 < q^2 < 15\mu^2$ (4.5). For $q^2 > 15\mu^2$ the model exhibits inadequacies, probably arising from the failure of (1.1a) to describe the radial dependence of the charge density in the internal regions of the proton structure, which are explored in electron-proton collisions at high momentum transfers. The goal of this note is to disclose the theoretical background presumably concealed behind the empirical model (1) and to outline a unified picture of the charge structure both of the proton and the neutron.

With this aim in view, let us show that the charge density (1.1a) can be deduced from the Poisson equation

$$\nabla^2 q_p(r) = 4\pi e g_p(r) ,$$

where the electron-proton potential $q_p(r)$ is a solution of the static, spherically

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