On an Approximate Solution to the Multichannel $N/D$ Equations.

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Summary. — The approximate solution to the multichannel $N/D$ equations proposed by Pagels is used to predict $S$-wave bound states in an attractive Yukawa potential. In the first-order Born approximation the results are reasonable, but large deviations occur when the second-order Born approximation is added to the potential. This raises a serious problem of how to take into account higher-order corrections to the potential.

1. — Introduction.

A method of reducing the partial-wave matrix $N/D$ integral equations to algebraic expressions has recently been given by Pagels (1). His resulting expression for the amplitude, which has the correct discontinuities on the right- and left-hand cuts, is symmetric, and independent of any subtraction points. Moreover, it is an algebraic function of the input potential. This last fact allows one to find restrictions on the potential necessary to produce bound states, resonances, or ghosts. All the other approximate solutions to partial-wave $N/D$ equations, for example, the determinantal method, have some limitations which are avoided in the Pagels' solution.

In this paper we would like to test the final expression for the amplitude and choose nonrelativistic potential theory as an example. We apply it to the case of $S$-wave bound states in a Yukawa potential, which is a single-channel calculation, and plot the bound-state energy as a function of the coupling constant. In the first-order Born approximation we get reasonable agreement with

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the results from the full $N/D$ integral equations given by Luming (2). The addition of the second-order Born term, however, does not improve the agreement between the results from the first-order Born term and the Schrödinger equation.

In Sect. 2 an expression is derived for the bound-state energy. The parameters we introduce are discussed in Sect. 3, where we also find the bound-state energy for the first and second Born approximations to the Yukawa potential. Our conclusions are contained in the final Section.

2. – Derivation of the bound-state energy.

In his paper Pagels used the $N/D$ equations as originally written down by Chew and Mandelstam (3). Most authors now use a form of the $N/D$ equations where a linear integral equation is first derived for the $N$-function, so we shall now show how to derive Pagels results. We assume that the following two equations are well known:

\begin{equation}
N(v) = B(v) + \frac{1}{\pi} \int_{\mathbb{R}} \left( \frac{B(v') - B(v)}{v' - v} \right) g(v') N(v') \, dv',
\end{equation}

\begin{equation}
D(v) = 1 - \frac{1}{\pi} \int_{\mathbb{R}} \frac{g(v') N(v')}{v' - v} \, dv'.
\end{equation}

Consider the case $v < 0$, and set $g(v') = -c v \delta(v' - a)$ in eq. (2.1) and eq. (2.2), leading to

\begin{equation}
N(v) = B(v) - c \cdot a N(a) \left( \frac{B(v) - B(a)}{v - a} \right)
\end{equation}

and

\begin{equation}
D(v) = 1 - \frac{c \cdot a N(a)}{v - a}.
\end{equation}

Taking the limit $c = a$ in eq. (2.3),

\begin{equation}
N(a) = \frac{B(a)}{1 + c \cdot a \left( \frac{\partial}{\partial v} B(v) \right)_{v=a}}.
\end{equation}

If we try to extend our result to the region $v > 0$ we immediately have a singularity in $D(v)$. Obviously we must transform eq. (2.4) into a derivative