Thermally Induced Residual Stresses in Eutectic Composites

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The effect of thermally induced residual stresses on the yield behavior of a unidirectionally solidified eutectic, (Co,Cr)-(Cr,Co)C2, is presented. At low temperatures, the yield stress is found to depend on the sense of the applied stress. The difference in yield stress between tension and compression is a function of temperature and disappears at a sufficiently high stress relaxation temperature. A straightforward analysis is presented that predicts the observed yielding behavior and a stress relaxation temperature that agrees well not only with the value obtained by observing the temperature dependence of the yield stress, but also with the value obtained from the thermal expansion behavior of the eutectic composite.

Residual stresses can develop in aligned fiber composites for a variety of reasons including thermal expansion mismatch, phase transformations and non-uniform plastic flow. Although the effect of such stresses on mechanical behavior has been recognized by Ebert et al. and others, no work to evaluate this effect in a directionally solidified eutectic composite has been reported. In this investigation, we examine the effect of thermally induced residual stresses on the mechanical behavior of the (Co,Cr)-(Cr,Co)C2 eutectic composite.

Residual Thermal Stresses

This section describes the effect of thermally induced residual stresses on the yield behavior of composites. Relations are derived which predict the magnitude of the residual stress in terms of measurable composite properties.

According to Laszlo, the residual stresses developed in a lamellar slab composite due to thermal expansion mismatch are

\[ \sigma_{R} = \frac{V_{2}M_{1}M_{2}}{V_{1}M_{1} + V_{2}M_{2}} \Delta \alpha \Delta T \]  
[1a]

\[ \sigma_{2}^{R} = \frac{V_{1}M_{1}M_{2}}{V_{1}M_{1} + V_{2}M_{2}} \Delta \alpha \Delta T \]  
[1b]

where \( M_{i} = \frac{E_{i}}{1 - \nu_{i}} \). These equations are approximate because it is assumed that stresses develop only in the plane of the lamellae, are constant across the thickness of a lamella, and are the same for every direction in the plane. A similar analysis can be applied to a fiber composite to show that the axial stresses are also given by Eqs.[1a] and [1b] except that \( M_{i} = E_{i} \) in this case, it is assumed that stresses develop only parallel to the fiber axis and are constant across each component of the composite.

Consider the case of a fiber composite cooling from the melt. We shall designate component 1 as the matrix phase, component 2 as the fiber phase, and assume that \( \alpha_{1} > \alpha_{2} \). Two regimes of behavior can be identified:

1) For \( T > T_{0} \), all stresses developed in the matrix and fibers upon cooling are relieved by creep, and

2) For \( T < T_{0} \), stresses develop in the matrix (tensile) and in the fibers (compressive), which increase linearly with decreasing temperature.

If a stress is applied to the composite parallel to the fiber direction, the resulting stresses in the matrix, fibers, and composite are related by the rule of mixtures

\[ \sigma_{c} = \sigma_{1}^{A} V_{1} + \sigma_{2}^{A} V_{2} \]  
[2]

If the fibers and matrix deform elastically, then

\[ \sigma_{1}^{A} = \sigma_{2}^{A} \frac{E_{1}}{E_{2}} \]  
[3]

The applied stress in the matrix is obtained by eliminating \( \sigma_{2}^{A} \) from Eqs.[2] and [3], which gives

\[ \sigma_{1}^{A} = \left( V_{1} + \frac{E_{2}}{E_{1}} V_{2} \right)^{-1} \sigma_{c} \]  
[4]

It is assumed that the fibers always remain elastic so that for \( T < T_{0} \), the proportional limit of the composite is reached when the sum of the applied and residual stresses in the matrix becomes equal to the proportional limit of the matrix, i.e.

\[ \sigma_{1}^{p} + \sigma_{1}^{A} = \sigma_{1}^{p} \]  
[5]

For this regime, Eqs. [1a] with \( M_{i} = E_{i} \) and [4] may be combined to give the proportional limit of the composite

\[ \sigma_{c}^{p} = \left( V_{1} + \frac{E_{2}}{E_{1}} V_{2} \right)^{-1} \left( \sigma_{1}^{p} + \frac{V_{2} E_{2}}{V_{1} E_{1} + V_{2} E_{2}} \Delta \alpha \Delta T \right) \]  
[6]

where the plus is for tension and the minus for compression and \( \sigma_{1}^{P} \) is always taken to be positive. With residual stresses, the proportional limit of the composite depends on the sense of the applied uniaxial stress. The difference between the proportional limit
in compression and that in tension is

$$\Delta \sigma_c^P = 2 V_1 E_1 \Delta \alpha_1 \Delta T$$  \[7\]

which decreases with increasing temperature becoming zero at the stress relaxation temperature.

The stress relaxation temperature can also be determined from the thermal expansion behavior of the composite. For $T < T_0$, the linear thermal expansion coefficient of the composite is given by

$$\alpha_c = \frac{V_1 \alpha_1 + V_2 \alpha_2}{V_1 \alpha_1 + V_2 \alpha_2}$$  \[8\]

For $T > T_0$, the matrix creeps and the thermal expansion coefficient of the composite approaches that of the fiber. The change in slope marks the stress relaxation temperature, which provides an independent check of the theory.

The preceding analysis can be extended into the plastic-elastic region with the result that the difference in flow stress between tension and compression is still given by Eq. [7] and is independent of strain. However, because of many effects that do depend on strain, this analysis is expected to be a good approximation only at very small plastic strains.

The thermally induced residual stress analysis presented here modifies the rule of mixtures in a straightforward manner. This analysis will now be shown to describe adequately the yield behavior of the unidirectionally solidified (Co, Cr)-(Cr, Co)$_7$C$_3$ eutectic. A more detailed but complicated analysis, which takes into account radial and tangential stresses, has been developed by Ebert and coworkers.

**PROCEDURE**

The (Co, Cr)-(Cr, Co)$_7$C$_3$ eutectic material used in this investigation was unidirectionally solidified from the melt in the form of 1/4 and 1 in. rods. Relatively slow solidification rates were employed so that the cast rods were, in effect, slow cooled from the melting temperature.

Tensile and compression specimens were machined from the directionally solidified rods with their stress axis parallel to the fiber axis. The tensile specimens had a gage section 2 in. diam. by 2 in. long. The compression specimens were rectangular parallelepipeds 0.1 in. sq by 0.3 in. long or cylindrical samples 0.125 in. diam. and 0.320 in. long. The compression data was the same for both of these two specimen shapes. The gauge length of the tensile specimens and the whole of the compression specimens were electropolished in a solution of 10 pct. perchloric acid in methanol prior to testing.

All tensile testing was performed in a Centorr high vacuum testing chamber mounted on an Instron universal testing machine. The crosshead speed was adjusted to give a strain rate of $3 \times 10^{-5}$ sec$^{-1}$. Test temperatures varied from room temperature to 1200°C and the temperature was controlled to ±3°C. All tensile tests were performed in vacuum at a pressure of approximately 10$^{-5}$ Torr. Compression tests were performed in a direct push compression fixture in air. Since all compression tests were at temperatures where the eutectic alloy forms a protective chromium oxide scale, no adverse effects on the mechanical properties would be expected during short-time compression tests in air. All specimens were soaked for 0.5 hr. prior to testing to minimize effects due to variation in heating rate.

The thermal expansion behavior of several of the specimens was determined up to 900°C employing a self-compensating quartz tube dilatometer.

**RESULTS**

Fig. 1 shows the structure of the unidirectionally solidified (Co, Cr)-(Cr, Co)$_7$C$_3$ eutectic, (Co, Cr)-(Cr, Co)$_7$C$_3$, with (a) transverse, and (b) longitudinal sections shown.

Thompson et al. have obtained stress-strain curves in tension and compression for the (Co, Cr)-(Cr, Co)$_7$C$_3$.