Time–Discretization of Nonlinear Systems with Time Delayed Output via Taylor Series

Zhang Yuanliang, Kil To Chong*
Faculty of Electronics and Information Engineering Chonbuk National University,
Duckjin-Dong, Duckjin-Gu, Jeonju 561–756, Korea

An output time delay always exists in practical systems. Analysis of the delay phenomenon in a continuous–time domain is sophisticated. It is appropriate to obtain its corresponding discrete-time model for implementation via a digital computer. A new method for the discretization of nonlinear systems using Taylor series expansion and the zero–order hold assumption is proposed in this paper. This method is applied to the sampled–data representation of a nonlinear system with a constant output time–delay. In particular, the effect of the time–discretization method on key properties of nonlinear control systems, such as equilibrium properties and asymptotic stability, is examined. In addition, 'hybrid' discretization schemes resulting from a combination of the 'scaling and squaring' technique with the Taylor method are also proposed, especially under conditions of very low sampling rates. A performance of the proposed method is evaluated using two nonlinear systems with time–delay output.

Key Words: Output Time–Delay, Scaling and Squaring Technique, Taylor–Series, Time–Discretization

1. Introduction

Time–delays associated with output measurements naturally arise in a variety of engineering applications. Indeed, one may consider cases where the process to be controlled or monitored is located far from the computing unit, the measured output data are transmitted through a low–rate communication system, or of sensor technology that inevitably introduces non–negligible time–delays, which when unaccounted for, may under mine the viability of the process control and monitoring system design. The convergence of communication and computation in control systems and the complex behavior of the control systems with non–negligible time–delays are the two main reasons for the special attention to the time–delayed status. It is difficult to apply the controller design technique developed during the last several score years for finite–dimensional systems to the systems with any time–delays in the variables due to their infinite–dimension. Thus, control system design methods which can solve the systems with time–delays are necessary.

A natural direction for time–delay system control is to attempt to extend the ideas and results of nonlinear non–delay control to systems with delay. Such results include the input–output linearization and decoupling, partial feedback linearization with delay term domination, and extension of control Lyapunov functions (CLF) to delay systems in the form of control Lyapunov–Razumikhin functions (CLR) [5]. Huang et al. (2004), presented a novel start–controlled phase/frequency detector for multiphase–output delay–locked loops. Gudvandén (1997) proposed a sliding Fermat number transform to reduce the input–output delay of finite ring convolvers and correlators. In

* Corresponding Author, E-mail: kitchong@chonbuk.ac.kr
Faculty of Electronics and Information Engineering Chonbuk National University, Duckjin–Dong, Duckjin–Gu, Jeonju 561–756, Korea. (Manuscript Received September 14, 2005; Revised April 12, 2006)
many applications magnetic levitation systems are required to have a large operating range. Choi and Baek (2002) applied Time Delay Control (TDC) to a single-axis magnetic levitation system to solve this problem. Germani et al. (2002) presents a new approach for the construction of a state observer for nonlinear systems when the output measurements are available for computations after a non negligible time delay. Lee and Kim (2003) proposed a high level CVT ratio control algorithm to improve engine performance by considering the powertrain response time delay. Cho and Park (2004) proposed a new impedance controller for bilateral teleoperation under a time delay.

Currently, modern nonlinear control strategies are usually implemented on a microcontroller or digital signal processor. As a direct consequence, the control algorithm has to work in discrete-time. For such digital control algorithms, one of the following time discretization approaches is typically used: time–discretization of a continuous time control law designed on the basis of a continuous time system; and time–discretization of a continuous time system resulting in a discrete–time system and control law design in discrete–time. It is apparent that the second approach is an attractive feature for dealing directly with the issue of sampling. Indeed, the effect of sampling on system–theoretic properties of the continuous–time system is very important because they are associated with the attainment of the design objectives. It should be emphasized that in both design approaches time discretization of either the controller, or the system model is necessary. Furthermore, notice that in the controller design for time–delay systems, the first approach is troublesome due to the infinite–dimensional nature of the underlying system dynamics. As a result the second approach becomes more desirable and will be pursued in the present study.

For digital simulation and design of continuous–time delayed systems, it is often required to have an equivalent discrete–time model available. In the field of the discretization, for the original continuous–time systems with time free case (Franklin et al., 1998), the traditional numerical techniques such as the Euler and Runge–Kutta methods have been used for acquiring sampled–data representations. However, these methods need a small sampling time interval. Due to physical and technical limitations slow sampling is becoming inevitable. A time–discretization method which expands the well–known time–discretization of the linear time–delay system (Franklin et al., 1998; Vaccaro, 1995) to a nonlinear continuous–time control system with time–delay (Kazantzis et al., 2003) can solve this problem. And this method is applied to the nonlinear control systems with delayed multi–input (Park et al., 2004a) and the nonlinear control systems with non–affine delayed input (Park et al., 2004b). Scaling and Squaring technique with this time discretization method can also be applied to the nonlinear control systems with delayed multi–input (Zhang and Chong, 2005).

In this paper, the digital state space representation of the dynamic systems with output time–delay is presented. The proposed discretization scheme applies Taylor Series expansion according to the mathematical structure developed for the delay–free nonlinear system (Kazantzis and Kvarais, 1997; 1999). In particular, the effect of the time–discretization method on key properties of nonlinear control systems, such as equilibrium properties and asymptotic stability, is examined. Also, the well–known “scaling and squaring” technique, which is widely used to compute the matrix exponential (Higham, 2004) is expanded to the nonlinear case when the sampling period is too large.

Following this introduction, Section 2 briefly describes the basic principles of discretization of nonlinear system with delay–free output. Section 3 presents the detailed discretization of nonlinear system with time delay output which is the main work of this paper, and Section 4 presents the scaling and squaring technique. Section 5 presents the computer simulations of the proposed algorithm. Finally, Section 6 presents a summary and the conclusions drawn from this study.